Historical Modeling of Risk-Bounded Buy-and-Hold Allocation

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Contents

Abstract and Summary

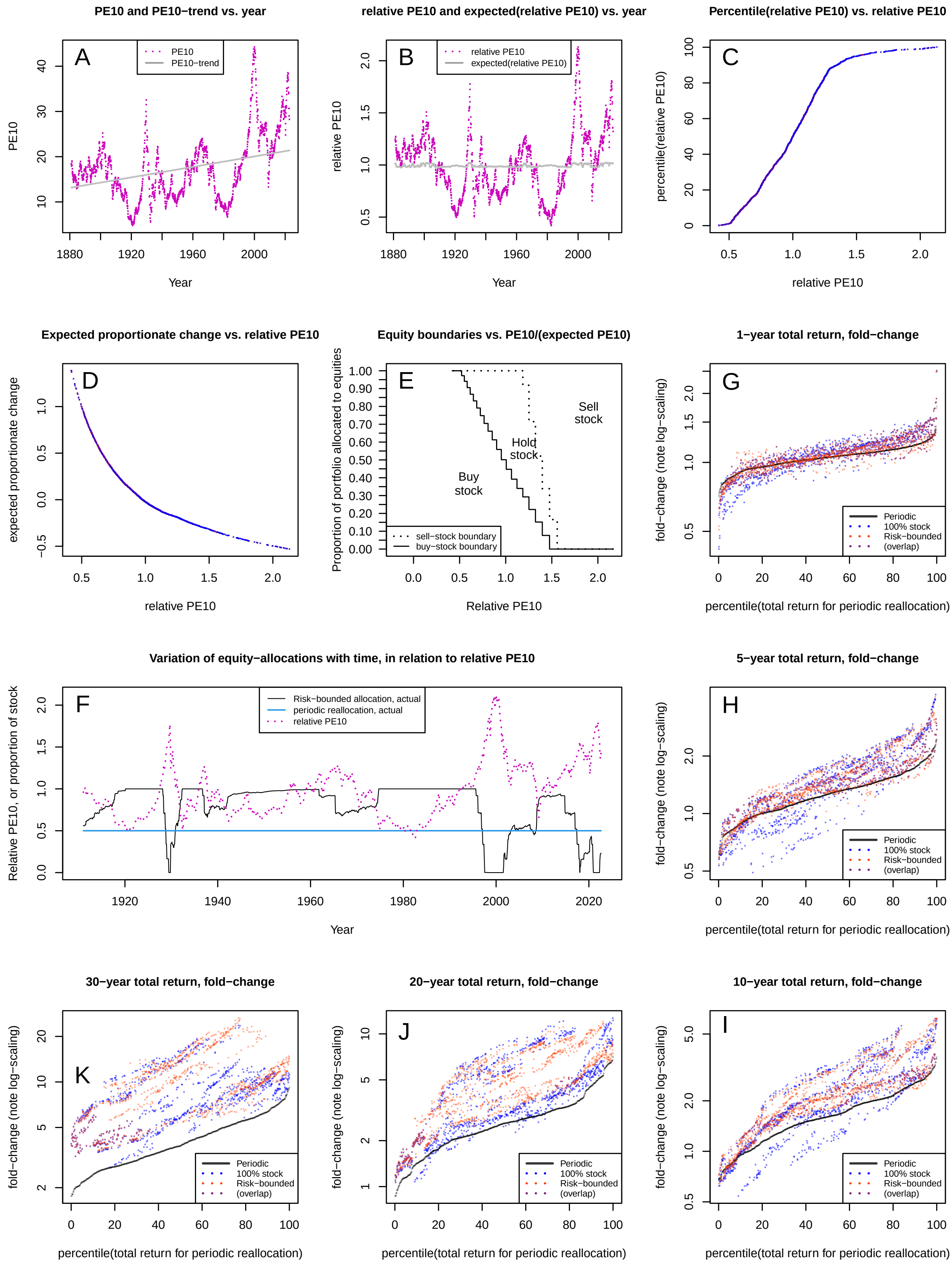
Since 1871, US stocks have produced greater "total returns" (dividends plus price change) than bonds in "real" (i.e., inflation-adjusted) dollars but have demonstrated much greater price volatility. One commonly recommended strategy to reduce the down-side volatility of a portfolio is to perform "periodic reallocation" (e.g., reallocating monthly, quarterly, or annually) to maintain a fixed percentage of stocks. An alternative to periodic reallocation (due to Benjamin Graham) is to adjust allocation incrementally in response to the price of stocks divided by an estimate of their "intrinsic annual earning power". Robert Shiller used the inflation adjusted earnings averaged over the preceding ten years to estimate the earning power of the S&P 500 stock market index; Shiller investigated the history of the ratio commonly named "the PE10" (i.e., the current price of the S&P 500 index divided by this estimate) to show that its increase is generally predictive of reduced total returns over the subsequent 20 years. Using Shiller's inflation-adjusted data for S&P 500 prices, earnings, and dividends and for US Treasury 10-year bonds, this study introduces a date-insensitive "relative PE10" and simulates the returns resulting from application of three allocation strategies: 60% stock "periodic reallocation", PE10-driven "risk-bounded buy-and-hold", and 100% stock buy-and-hold. From 1911 through 2022, over time spans of $\geq$ 5 years, the "risk-bounded buy-and-hold" strategy generally would have produced greater annualized real returns than the 100% stock allocation with less down-side volatility, while the 60% stock "periodic reallocation" strategy would have produced significantly lower returns with only moderately less down-side risk.

The simulation procedure used here, its underlying reasoning, and the overall results are summarized in the following table and figure:

- From 1881 through 2022, the PE10 has slowly increased {panel A in figure}; a date-insensitive "relative PE10" was derived as $\frac{\text{PE10}+max(\text{PE10})-trend(\text{PE10})}{median(\text{PE10})}$ such that 1.0 is the median value {panel B}.
- When the relative PE10 is percentile ranked {panel C}, the upper half of the range is at or above the 90th percentile; at this point, statistically, the PE10 and index price are not expected to be sustainable.
- The cost (or benefit) of the relative PE10 reverting to 1.0 was computed {panel D}.
- Guided by the cost of reversion to the median, minimum and maximum "risk boundaries" (respectively, "buy stock" and "sell stock") were assigned to each percentile-range of the relative PE10 {panel E}.
- For "risk-bounded buy-and-hold" (apart from reinvestment of dividends), stocks were traded incrementally as their assigned risk boundaries were crossed {panel F}.
- For "periodic reallocation", the portfolio was rebalanced monthly to 60% stock.
- Total returns were plotted (versus percentile of the total returns from the starting month for "periodic reallocation") as fold-change in portfolio value over 1-year time-spans {panel G}; 5-year {panel H}; 10-year {panel I}; 20-year {panel J}; 30-year {panel K}. (Blue dots atop orange ones are purple.)
- Downside deviation {table below} was calculated as $\frac{\text{first quartile}}{\text{second quartile}} - 1$, i.e., 75% of values are higher (roughly analogous to one standard error below the mean for a log-normal distribution).

"Buy as the bears appear; hold while the air is clear; sell as stocks get too dear."

Panel	Year-span	Risk-bounded returns, annual	Rebalanced returns, annual	Stock returns, annual	Bond returns, annual	Risk-bounded downside, annual	Rebalanced downside, annual	Stock downside, annual	Bond downside, annual
G	1	7.48%	5.26%	8.72%	1.96%	-9.2%	-7.6%	-11.6%	-5.7%
H	5	7.09%	4.76%	7.54%	1.46%	-4.1%	-4.0%	-7.1%	-2.6%
I	10	7.97%	4.86%	7.05%	1.41%	-3.2%	-2.6%	-3.7%	-2.3%
J	20	8.10%	4.87%	6.86%	2.27%	-2.1%	-1.4%	-2.2%	-2.7%
K	30	7.78%	4.53%	7.07%	1.79%	-1.4%	-0.9%	-1.2%	-2.0%
	81	7.40%	4.51%	6.97%	1.72%	-0.3%	-0.2%	-0.5%	-0.2%


PE10 and PE10−trend vs. year
A
PE10
PE10−trend
Year
relative PE10 and expected(relative PE10) vs. year
B
relative PE10
expected(relative PE10)
Year
Percentile(relative PE10) vs. relative PE10
C
percentile(relative PE10)
relative PE10
Expected proportionate change vs. relative PE10
D
expected proportionate change
relative PE10
Equity boundaries vs. PE10/(expected PE10)
E
Proportion of portfolio allocated to equities
Sell stock
Hold stock
Buy stock
sell−stock boundary
buy−stock boundary
Relative PE10
1−year total return, fold−change
G
fold−change (note log−scaling)
Periodic
100% stock
Risk−bounded
(overlap)
percentile(total return for periodic reallocation)
Variation of equity−allocations with time, in relation to relative PE10
F
Relative PE10, or proportion of stock
Risk−bounded allocation, actual
periodic reallocation, actual
relative PE10
Year
5−year total return, fold−change
H
fold−change (note log−scaling)
Periodic
100% stock
Risk−bounded
(overlap)
percentile(total return for periodic reallocation)
30−year total return, fold−change
K
fold−change (note log−scaling)
Periodic
100% stock
Risk−bounded
(overlap)
percentile(total return for periodic reallocation)
20−year total return, fold−change
J
fold−change (note log−scaling)
Periodic
100% stock
Risk−bounded
(overlap)
percentile(total return for periodic reallocation)
10−year total return, fold−change
I
fold−change (note log−scaling)
Periodic
100% stock
Risk−bounded
(overlap)
percentile(total return for periodic reallocation)

Note that this is a "living document"; I am working on rephrasing it over time. I welcome any suggestions to make it clearer.

Executive summary

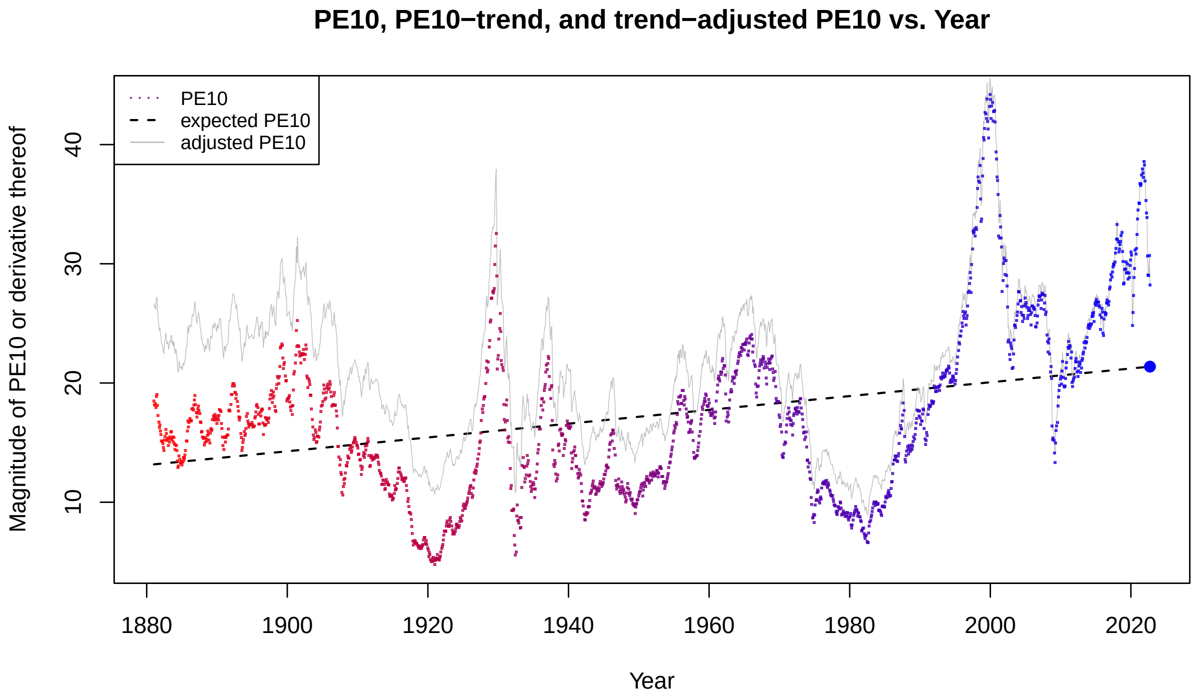
A frequently recommended strategy for retirement investing calls for maintaining a fixed ratio of stocks to bonds in a portfolio. Such strategies recommend stock holdings to generate income (from stock dividends) and bond holdings (to buffer the overall volatility of the portfolio, so that some of the value of the principal will be protected when withdrawals are required), with regular reallocation to maintain the ratio between the two (adjusting according to fluctuation in the prices of stocks and bonds). The intent (I believe) is to allow the investor to "set it and forget it" without the need to pay close attention (or even understand what is happening), except for regularly rebalancing. Unfortunately, these strategies ignore important costs that they incur. First, frequent trading (by either investors or fund managers) is required to maintain alignment of the percentage with the market. Second, while stock dividends are generous when the price is high, they are not always high enough to compensate for the loss in the value of the principal when the price reverts from the extreme. Third, this strategy is only minimally effective at buffering volatility.

A Graham-based strategy for determining the proportion of stocks in a portfolio considers the risk of these costs; rather than trading to meet an arbitrary percentage, the strategy calls for selling stock only when the likely cost of holding stock is too high and for buying stock only when it is likely that investing in stock will give good returns in the long term. It "values" stock based on its present price and its long-term earning power, using the value as a criterion for when trading is (infrequently) needed. A simple representation of such a valuation metric is the ratio of the price to the ten-year moving average of its earnings ("the PE10"). (Although, theoretically, more complicated projections of the future earning power may be performed, these are nevertheless too uncertain to consider here.) Note also that, buy using broadly diversified index funds, portfolio volatility may be decreased by overperformers offsetting underperformers.

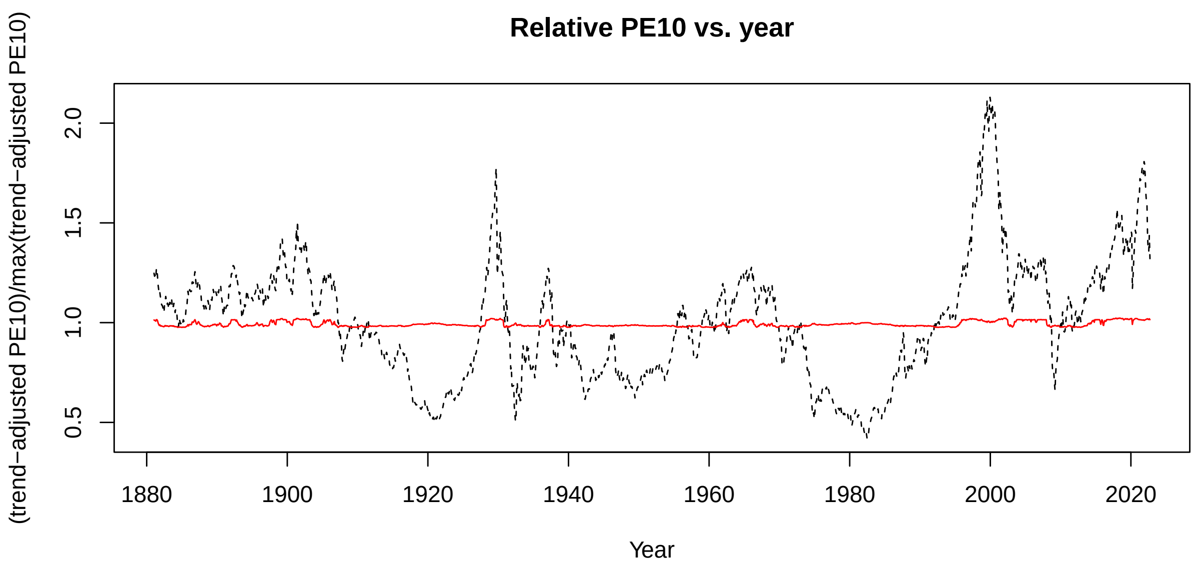
Have total returns (dividends plus capital appreciation) been highly correlated with the PE10? Before considering whether the PE10 has utility in planning an investment strategy, we need to look at how it has varied in relation to the performance of the S&P 500.

The first figure illustrates the following transformations of the PE10, i.e., the actual price divided by the average earnings over the preceding ten years:

- The PE10 itself, for which earlier points are colored more red; intermediate, more purple; later, more blue.
- The "expected" PE10, i.e., a value projected from a trend fitted to the actual PE10's over the history of the S&P500
  - The trend fit is a "Tukey line" with five iterations, as described in the body text.
  - This is an estimation of the "center" for the S&P 500 index price at each time; 851 points are above it and 850 points are below it.
  - This trend is entirely contrived; it only represents reality exactly when the PE10 crosses it.
- The maximum expected PE10, shown as a blue dot
  - This is the most recent value of the expected PE10.
- The "adjusted" PE10, i.e., the PE10 plus the maximum expected PE10 minus the expected PE10
  - To obtain values that are comparable without date-sensitivity, this adjusts the PE10 so that the maximum expected PE10 is the median of the adjusted PE10 at any time.
- The "relative" PE10, i.e., the adjusted-PE10 divided by the median adjusted-PE10
  - This expresses the deviation of the PE10 as a proportion of the PE10 in a date-insensitive manner.



The statistically expected relative-PE10, computed as described in the section of the Appendix labeled "Figure A3 - Expected relative-PE10 vs. relative-PE10", is plotted (as a solid red line) vs. year in the next figure along with the relative-PE10 (as a black dashed line).



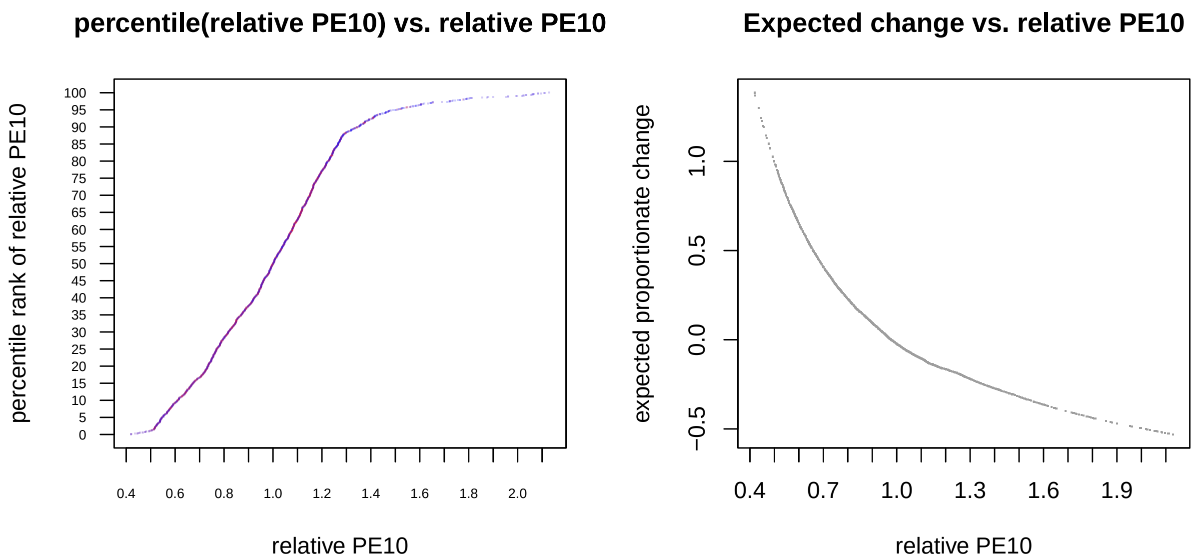
From PE10 to risk-bounded equity proportion

The relative PE10 provides a metric for downside risk: its value represents the (proportionate) "cost" of returning to the median value, and its percentile represents the "probability" that it will decrease. See

Altruist (2022) for references to formal discussions of downside risk.

The left panel of the next figure shows that the PE10 (relative and otherwise) does not remain at the upper extreme for long but it can be disproportionately high; this is what a bubble looks like. Thus, when the PE10 was extremely high, a market correction was both inevitable and costly. At most, the portfolio occasionally lost more than twice its value (although that was rare).

The right panel estimates the expected proportionate loss when the relative PE10 is above one. This is computed as described under "Figure A4 - Expected proportionate change vs. relative-PE10" in the Appendix.

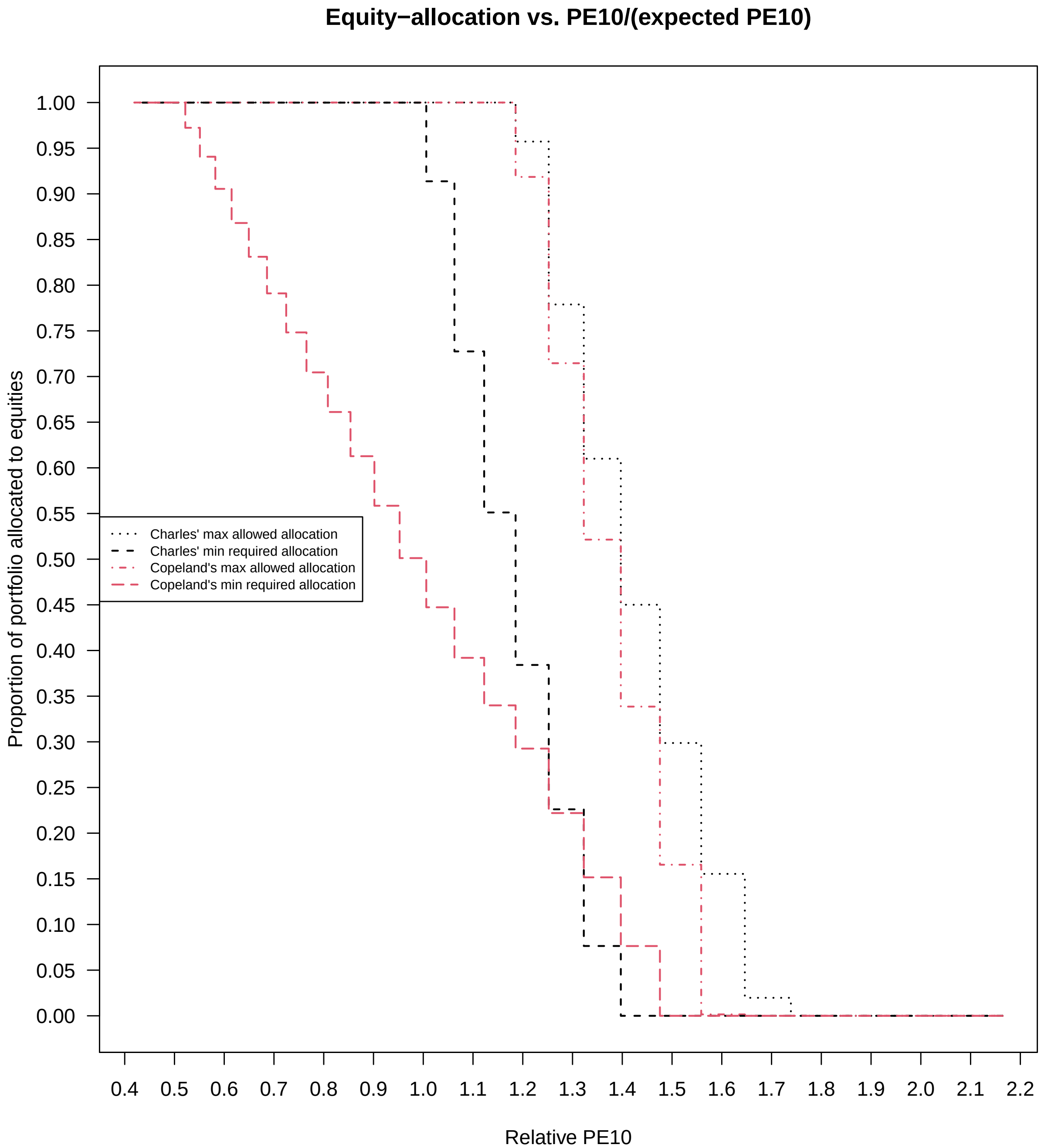


Could the expectd change in the PE10 be used to assess risk and set limits to investment in the S&P500? I tested this against the historical record, using data extracted from the *ie_data.xls* spreadsheet Shiller (2023).

On the basis of the PE10, the two Graham-based strategies presented here ("Copeland's (Co)" and "Charles' (Ch)") dictate:

- buy stock to the minimum required proportion of stocks in the portfolio when the PE10 is low, and
- sell stock the maximum allowable proportion of stocks when the PE10 is (too) high relative to its expected value.
- do not trade when the PE10 is between the minimum and the maximum (i.e., allow the stock allocation to drift).
  - See French (2021) to get an intuitive feel why not rebalancing provides better returns than rebalancing.
  - It also implies that the there considerable time can between when purchases and sales are recommended.

These are illustrated in the next figure; see the section of the body text entitled "A risk-bounded buy-and-hold allocation strategy" for an explanation of how the allocations are calculated. The do-not-trade zone for Copeland's strategy is the boot shaped area between the pairs of red curves; for Charles' strategy, the roughly trapezoidal area between the black curves.



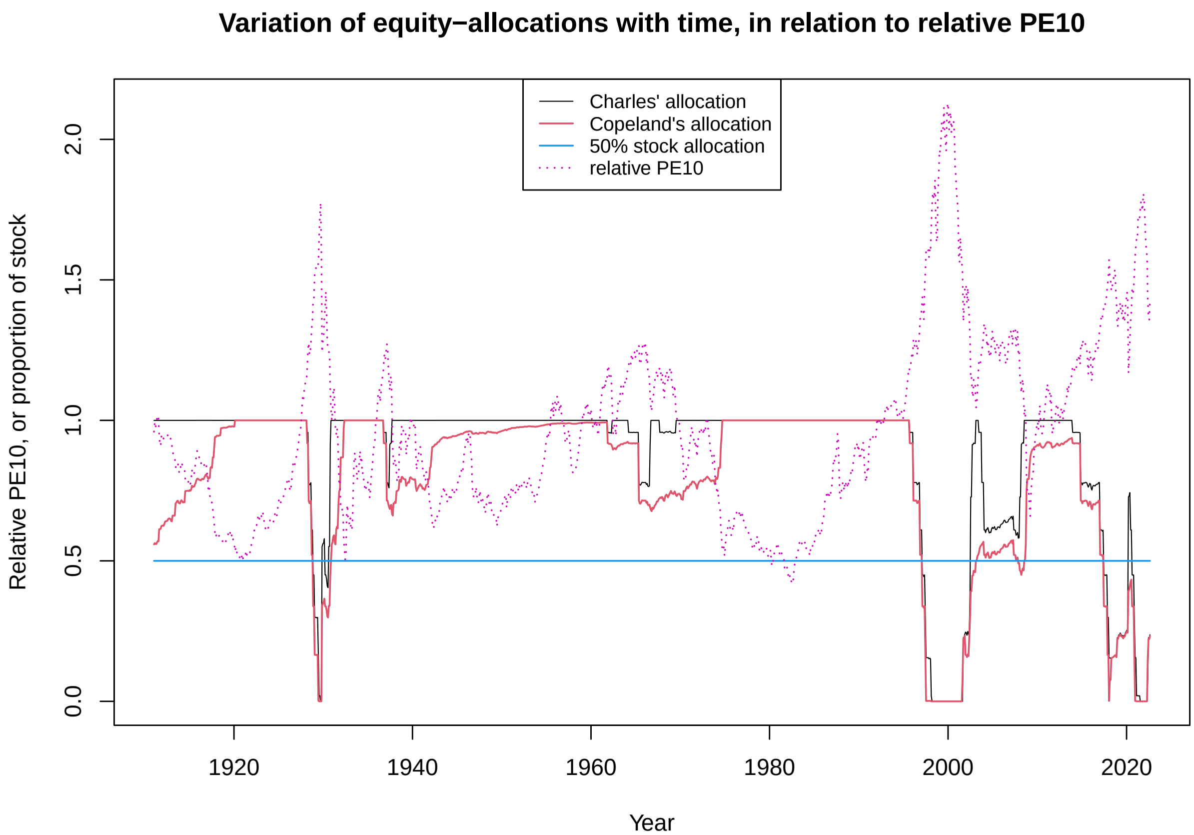
Notably, both Charles' and Copeland's strategies reach their minimum stock allocation when the relative PE10 is the 97$^{\text{th}}$ percentile and above, i.e., when the relative PE10 has reached 1.7.

As discussed in the body text, there is a theoretical difference between Copeland's and Charles' strategies. Copeland's is oriented toward "downside risk", i.e., risk of loss if stock prices revert to their expected values. Charles' is oriented toward minimizing volatility, i.e., uncertainty regarding the future price of securities, reducing expected volatility when stocks are overvalued. Although these are technically different, with appropriate adjustment, they can converge to produce very similar results.

Variation of the investigated allocations with time

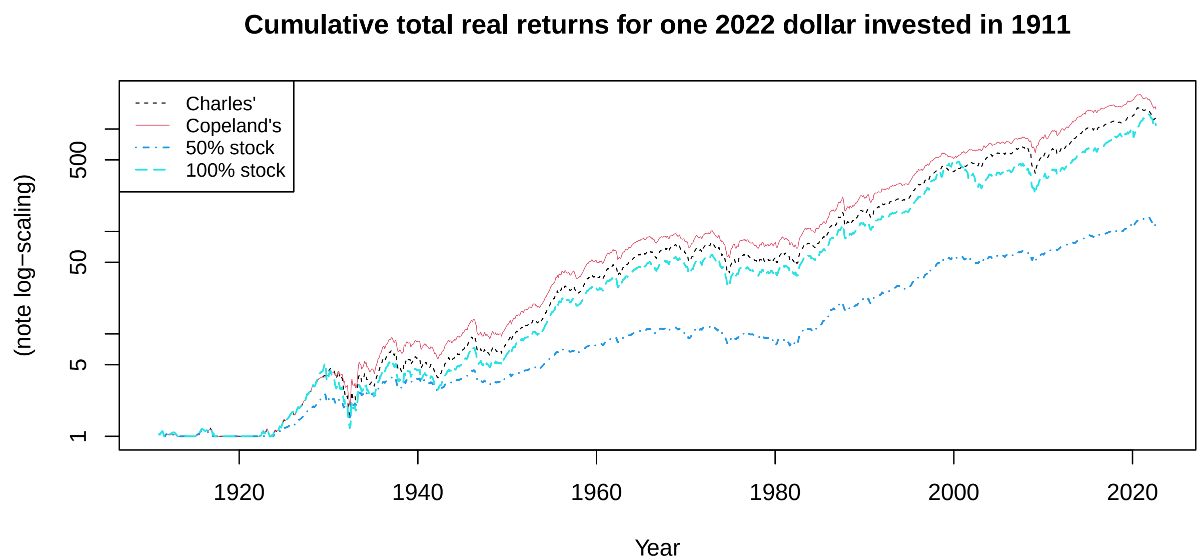
The next figure presents how the allocation-strategies presented above would have been applied for the history of the S&P 500 stock index for the period 1911–2022.

This shows, again, that the Graham-inspired strategies would have required minimizing stock allocation only when the relative PE10 was extremely high (97^{th} percentile and above).



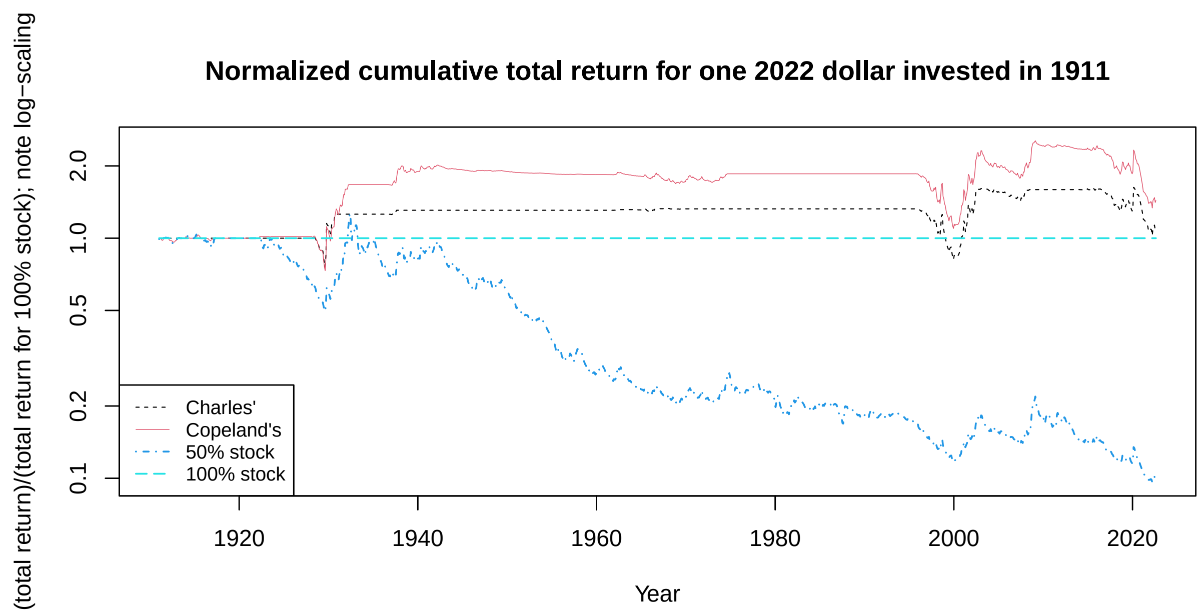
Total real returns for each strategy, for 1911–2022.

The next two figures show that both Charles' and Copeland's allocation strategies would have substanially outperformed the 100% equity allocation (using "real", i.e., inflation-adjusted, dollars). Note that displacement along the Y-axis is proportional to log(total return) because total return grows geometrically.



Cumulative total real returns for each strategy, relative to 100% stock allocation.

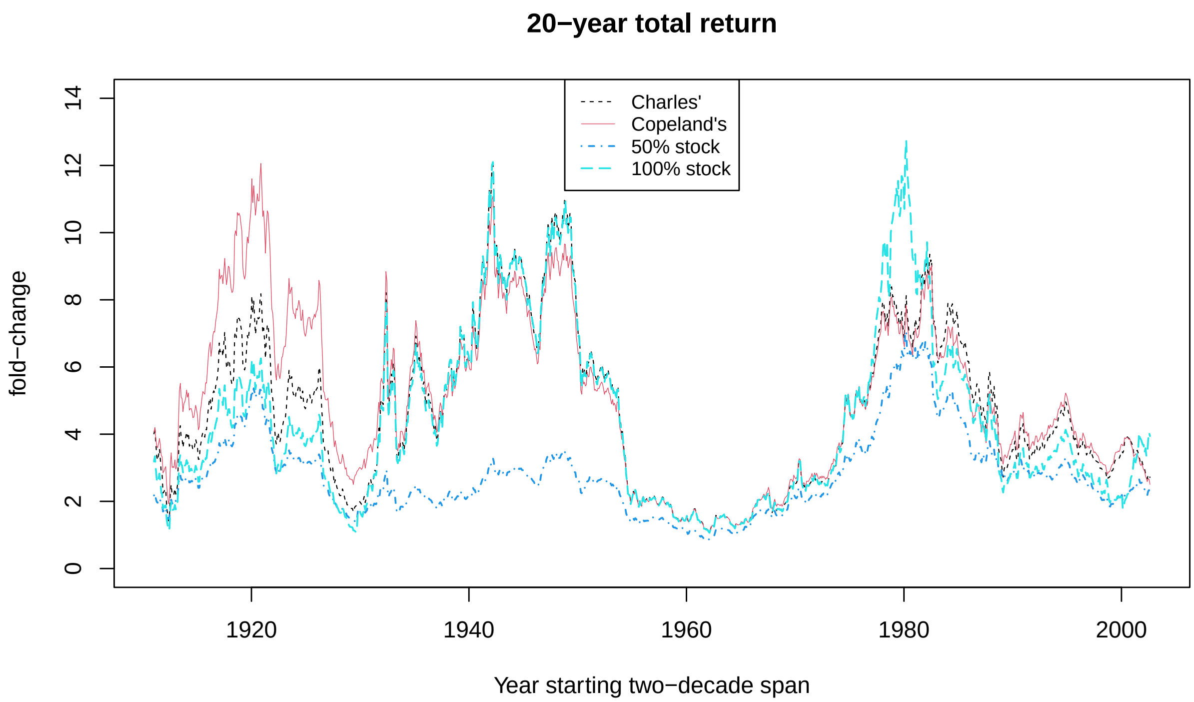
To make the relative effect of the several allocation strategies more readily apparent, the data above are plotted here after dividing the return of each allocation by the return of the 100% stock portfolio. (Note that the 50% stock portfolio would have been producing positive net returns overall; because of the transformation, it may appear to be decreasing when, in fact, it is merely falling behind the 100% stock portfolio.)



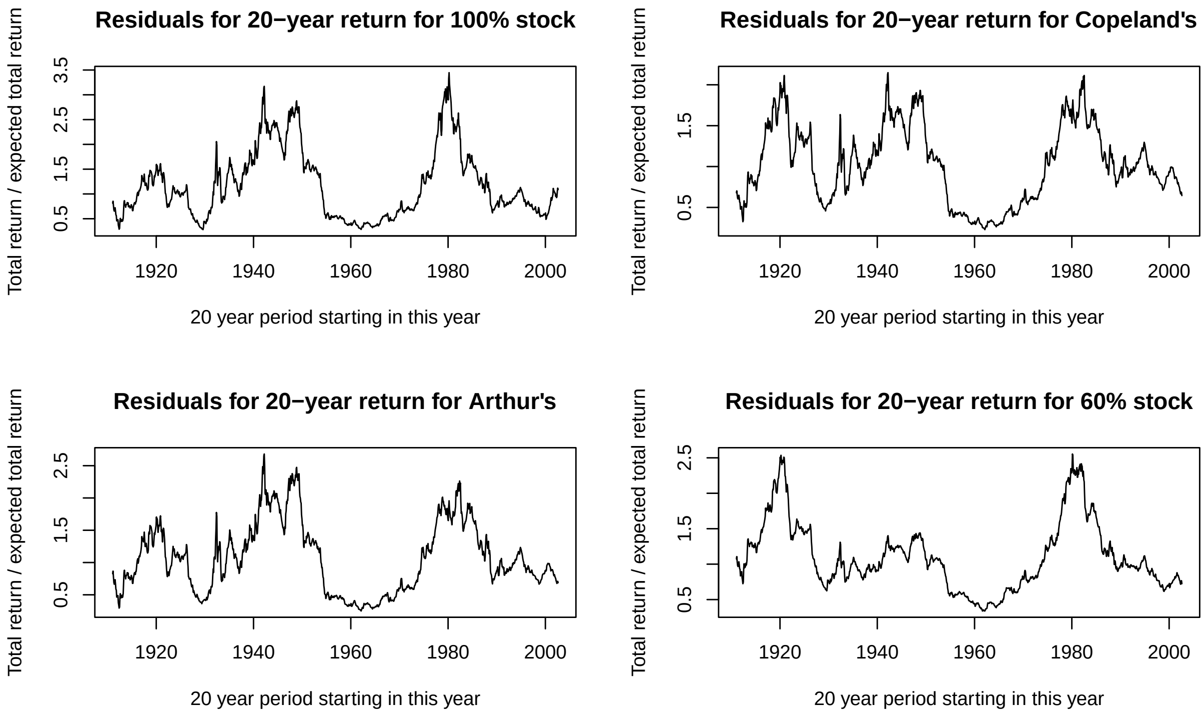
All three alternatives to the 100% stock allocation would have performed much better during the market turmoils of 1929, 2000, 2008, and 2020, i.e., they move upward relative to the 100% stock portfolio. These are cumulative returns; how might returns compare over a fixed time horizon?

20-year total real returns from 1911 through 2022

The next plot shows 20-year total returns for each strategy. Copeland's returns are higher than Charles'; Charles', than 100% stock.



The next figure shows the deviation of the 20-year total returns for each strategy from its expected value (computed as Tukey-lines with five iterations). These differ primarily in timing of the peaks. Over 20 years, all three strategies reduce relative volatility by about 20%.



Real returns from 1911 through 2022, by allocation strategy

The following table shows the effective growth rates for each of the strategies.

Allocation strategy	Total return	Annual growth	Required trades
Copeland's	$1562.33	6.81%	171
Charles'	$1200.12	6.56%	177
100% equity	$1088.06	6.47%	0
50% equity	$108.58	4.29%	1339
0% equity	$5.49	1.54%	0

Fixed-percentage stock allocations, at any level (not presented here here) could not outperform a 100% stock allocation. By contrast, the risk-bounded buy-and-hold Graham-based allocation can offer an increase in real returns, requiring minimal trading on the part of the investor.

Not shown here:

- Even if Copeland's risk-bounded buy-and-hold were capped at 60% stock, it still would have generated an average of 0.24 more percent annual returns than the fixed 60% stock portfolio, with fewer months where trading was required.
- Even if the trend in the PE10 were computed for values only through February, 2003, Copeland's risk-bounded buy-and-hold would have generated only 0.15 less percent annual returns than when the trend was computed through October 2022 (i.e., 7.05% vs. 7.22%).

Motivation

I want to have a financial plan for retirement in which I can have confidence. For this document, my concern is generating income from savings.

I want us to invest our savings such that we will generate sufficient income indefinitely, i.e., for 35 years or more. This means that, on average, it must produce more income than our expenses. The simplest way to do that, of course, is to produce the most sustained income that we can buy.

Adjusting for inflation, US large-cap stocks can produce about three times the inflation-adjusted income that bonds can produce, albeit with much more price volatility. Over the long term, a considerable allocation into stocks seems like a wise strategy. I want our income strategy to be reasonably simple, avoiding unnecessary trades and the costs of money, time, and attention that they incur. I want a (mostly) "set it and forget it except at rare moments of opportunity (or peril)" plan. So, for the most part, I would like to "buy and hold **and collect income** from stocks".

I reason that it usually makes sense to hold stock for as much of the time as possible, not worrying about the price because I would not be intending to sell it except under extraordinary circumstances. If stocks are generating enough income, such circumstances would be to sell only when stocks are grossly over-valued and only gradually (if ever) to meet expenses. I want to hold only enough bonds to buffer the stocks' volatility until such time as their price has risen to some (hopefully optimal) level.

If the strategy is purely to buy and hold, then there is a linear relationship between how much is paid and the per-retirement-dollar income that is generated. If one pays slightly too much for stock, the price difference can be made up in a short time by the increased earnings relative to the earnings of bonds. On the other hand, if the price is far too high, it can take a very long time for earnings to pay back for the difference - effectively, this looks identical to a reduced amount of money that has been invested.

Note also that I would like to keep my investment strategy (nicknamed "Copeland's" here) simple, so I will also constrain my analysis to allocation between 100% stocks (i.e., the composition of VTWAX, Vanguard Total World Stock Index Fund, Admiral Shares) and 0% stocks (i.e., the composition of VTINX, Vanguard Target Retirement Income fund). To minimize volatility, these funds are as broadly diversified as the total domestic and international markets for stocks and bonds; the two differ mainly in the percent allocation to stocks. Of course, the analysis here would chiefly reflect the domestic portions of these funds.

This leaves open two questions:

- On average, how much income can stocks generate?
- What determines when to buy, and how much?

To address these questions, I investigated how a few different allocation strategies would have fared over the past 150 year history of the S&P 500, both in terms of how much income they generate and how purchase price for stocks impacts the return on investment.

Introduction

Graham and Dodd (1934) and Shiller (2000) build the case for avoiding investment in stocks when their price is excessively high.

Historical returns of S&P 500 and Ten-year US Treasury bonds

One metric to establish relative value of a stock (or the market) is the price-to-earnings ratio (P/E). Inflation affects the numerator and denominator of P/E (almost) equally, suggesting that the P/E might be comparable across the historical record.

Over the period 1911-2022, the average annualized, "real" (i.e., inflation-adjusted) total return of the S&P 500 index (used here as a proxy for a broadly diversified portfolio of USA stocks), assuming reinvestment of all dividends, was 6.44%, whereas the return 10-year US Treasury bonds (also known as "GS10"; used here as a proxy for medium-to-long term bonds), assuming reinvestment of all dividends, returned 2.02% per year during the same period.

It seems clear that, over the long term, it is most sensible to allocate as much as possible toward stocks away from bonds, taking into account stocks' price volatility. How can one estimate how much the price of stocks is expected to change, over the long term?

CAPE (PE10): the Cyclically Adjusted Price to Earnings Ratio

Campbell and Shiller (1988) defined the "Cyclically Adjusted Price to Earnings Ratio" (CAPE), commonly known in the value-investing community as the PE10, as the current price of the S&P 500 stock index divided by the average inflation-adjusted earnings over the preceding ten years. They wrote:

> "Graham and Dodd (1934) recommended an approach that 'shifts the original point of departure, or basis of computation, from the current earnings to the average earnings, which should cover a period of not less than five years, and preferably seven to ten years.' (*Security Analysis*, page 452)."
>
> "These results ... establish that a very high proportion of multiperiod returns is forecastable using a long moving average of earnings."

Demonstration: Risk-bounded buy-and-hold equity allocation strategy

I will present below some graphs showing why I take the PE10 to be a reasonable metric upon which to base a risk-bounded percent-equity allocation in a buy-and-hold oriented strategy, along with some examples that would have weathered favorably the dips in the S&P 500 index price during 1904, 1907, 1929, 2008, and 2020.

Considering allocation between the S&P 500 stock index and 10-year Treasury bonds to mitigate the risk to the portfolio of PE10 returning to the expected value (and ignoring the risk of it going below), the results presented below are for an estimated minimum and maximum for this risk, respectively, of 1.00% and 38.90%.

Note: Because one cannot invest directly in an index, I substituted VTSAX (Vanguard Total Stock Market Index Fund, Admiral shares) for the S&P 500, with an expense ratio of 0.00% per year; for the GS10, I substituted VSIGX (Vanguard Intermediate-Term Treasury Index Fund, Admiral shares, with an expense ratio of 0.07% per year.

Background

Data for this analysis

Data source

Each month's data for the historical S&P 500 performance and for Ten-Year Treasury yields since 1911, along with the computation for PE10 (CAPE), were taken from columns "A" through "V" of the "Data" tab from the "alternate version of CAPE" spreadsheet *http://www.econ.yale.edu/~shiller/data/ie_data.xls* Shiller's webpage (*http://www.econ.yale.edu/~shiller/data.htm*).

Data transformation

Data transformations were performed in R using file `HistoricalReturByAllocationStrategy.Rmd` from which the document that you are reading was produced. It may be found at https://eschenlauer.com/investing/risk_based_allocation

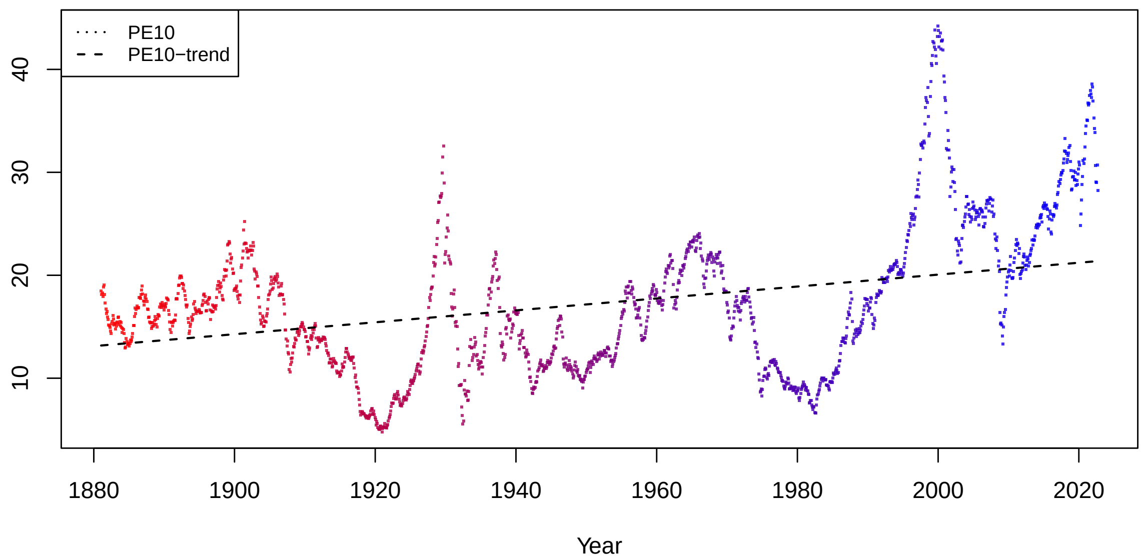
Not yet incorporated in the model

- Calculation and consideration of percent uncertainty (a.k.a. "investment risk").
  - Note that percent uncertainty decreases with $\sqrt{\text{timeline}}$ (very roughly)

PE10 has been slowly increasing

The first figure shows that, over time, the PE10 has been increasing. Points from earlier years are more red; from later years, more blue; from middle years, more purple.

The prediction line is Tukey's robust line fitting (Tukey, 1977), as implemented in the `stats` package in R, `stats::line`, run for five iterations.



This trend was noted by Bernstein (2013), who philosophized regarding its interpretation.

Trend-adjusted PE10

To remove the effect of the trend, the PE10 may be adjusted by applying Tukey's robust line fitting (run for five iterations) to its trend and subtracting the result:

$$\text{adjusted-PE10}_i = \text{PE10}_i + \text{expected adjusted-PE10} - \text{PE10-trend}_i$$

where i is the month number and

expected adjusted-PE10 = max(PE10-trend) = 21.3748745 for October 2022.

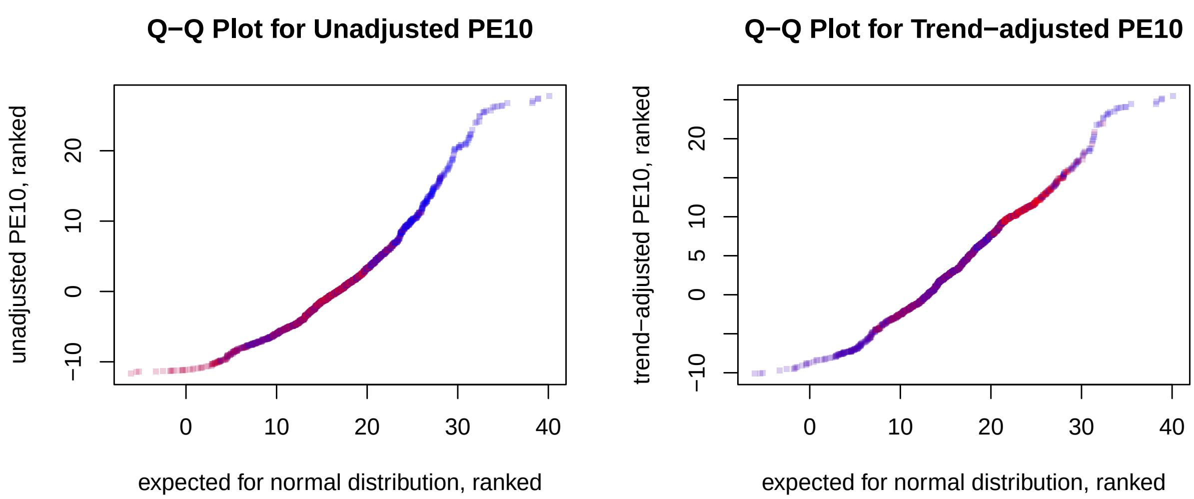
and

PE10-trend$_i$ = (0.0578777)(DateFraction) - 95.694931

Although a logarithmic fit could be employed, it would not look much different and seems like a needless complication to introduce. (Indeed, when this is done, it is visually indistinguishable and at most about 3% different.)

Does removing the trend seem to make sense?

What is the effect of adjusting for the trend? This is explored by plotting the differences between the observations and the trend (i.e., "the residuals") versus what would be expected for a normal distribution. Again, the earlier values are more red; the later, more blue. (Bear in mind that there is no reason to assume normality!)

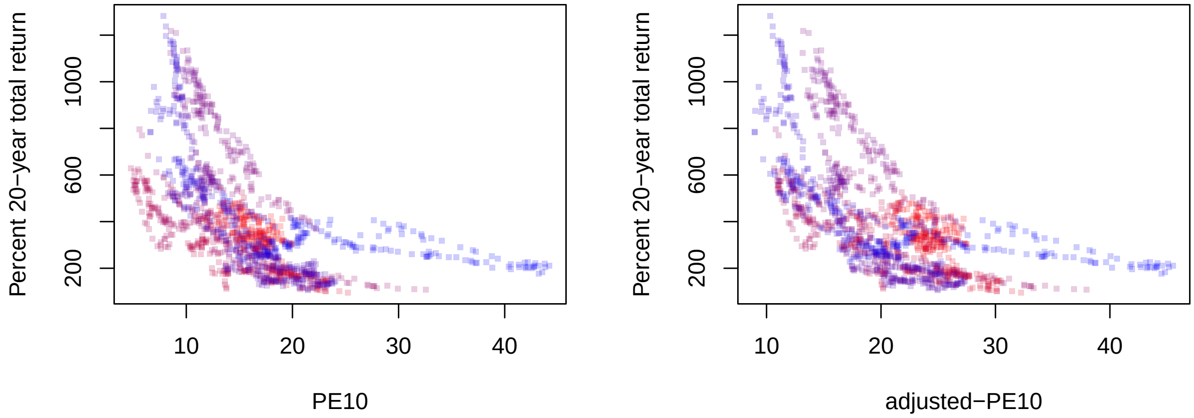


The gentle curve in the center of the range has been removed by the adjustment procedure.

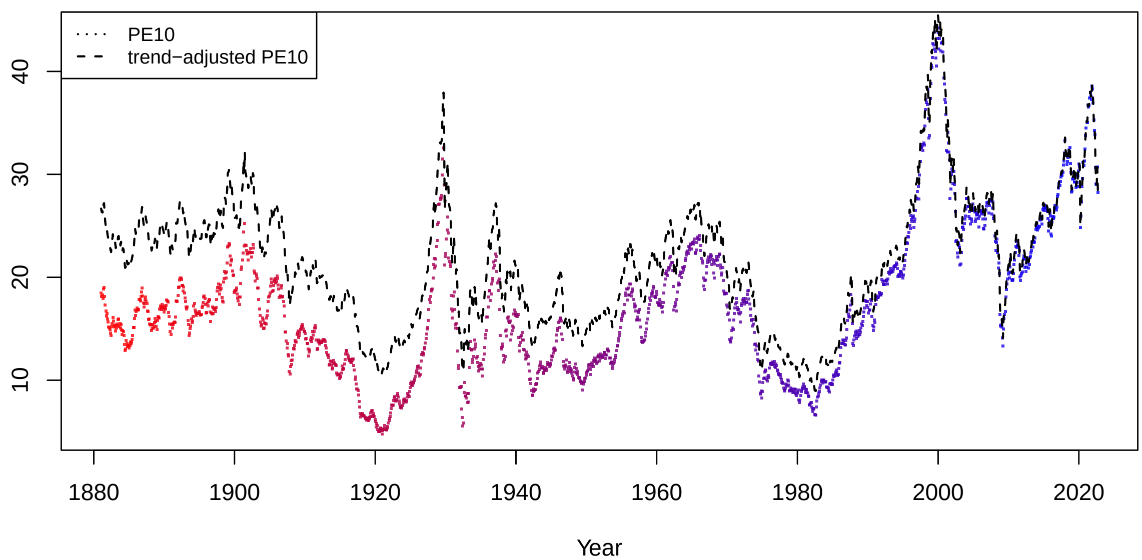
PE10 is predictive of 20-year total return

The left panel of the next figure presents a plot of the twenty-year total return versus the PE10, showing clearly that, as PE10 rises, the potential for return falls. Points from earlier years are more red; from later years, more blue; from middle years, more purple.

The right panel shows that using the adjusted-PE10 tightens the spread of the points, by moving early (red) points up and to the right:



Next are shown the unadjusted and adjusted PE10 on the same plot:



Although I am not demonstrating it here, the trend-adjustment is helps stabilize the risk estimation discussed below.

Relative PE10

Both for calculations below and for intuitive thought, it is convenient to define a "relative PE10" as:

$$\text{relative PE10}_i = \frac{\text{adjusted-PE10}_i}{\text{expected adjusted-PE10}} \approx \frac{\text{adjusted-PE10}_i}{median(\text{adjusted-PE10})}$$

Hence:

$$median(\text{relative PE10}) = 1.0$$

As shown in the section of the Appendix labeled "Figure A4 - Expected proportionate loss vs. relative PE10",

$$\text{expected relative PE10} \in 1.0 \pm 2.5\% \approx \textit{median}(\text{relative PE10})$$

Accounting for the risk that the adjusted PE10 will return to the median

Note that by "risk", here, I mean the cost of the adjusted PE10 returning to the expected expected, expressed as a proportion of the current adjusted PE10.

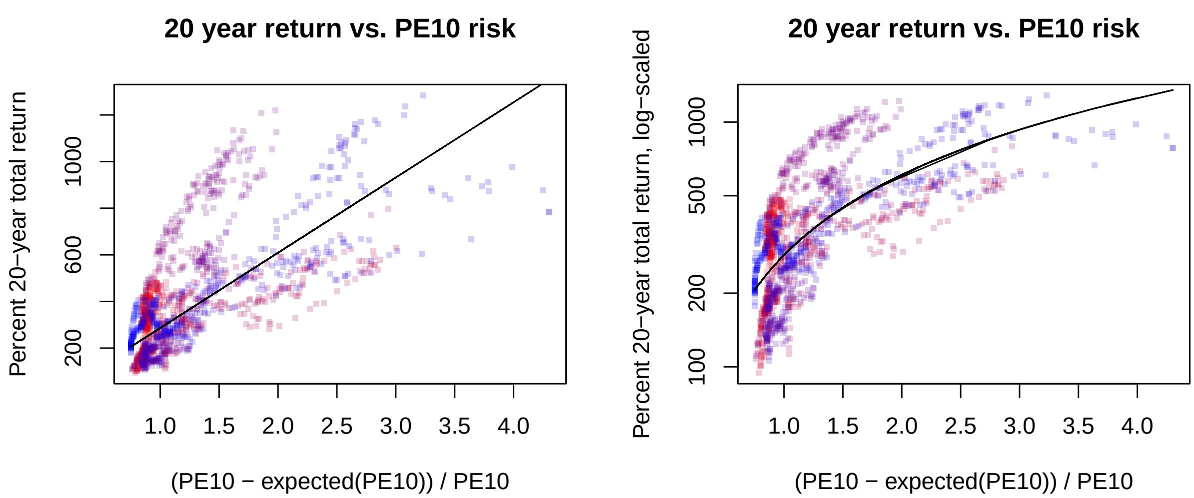
One may define the "PE10-risk" as of month i returning to the PE10 trend, accounting for the linear trend in PE10, as:

$$\text{PE10-risk}_i = \frac{\text{PE10}_i - \text{expected-PE10}_i}{\text{PE10}_i} = 1 - \frac{\text{expected adjusted-PE10}}{\text{adjusted-PE10}_i} = 1 - \frac{\text{expected relative PE10}}{\text{relative PE10}_i}$$

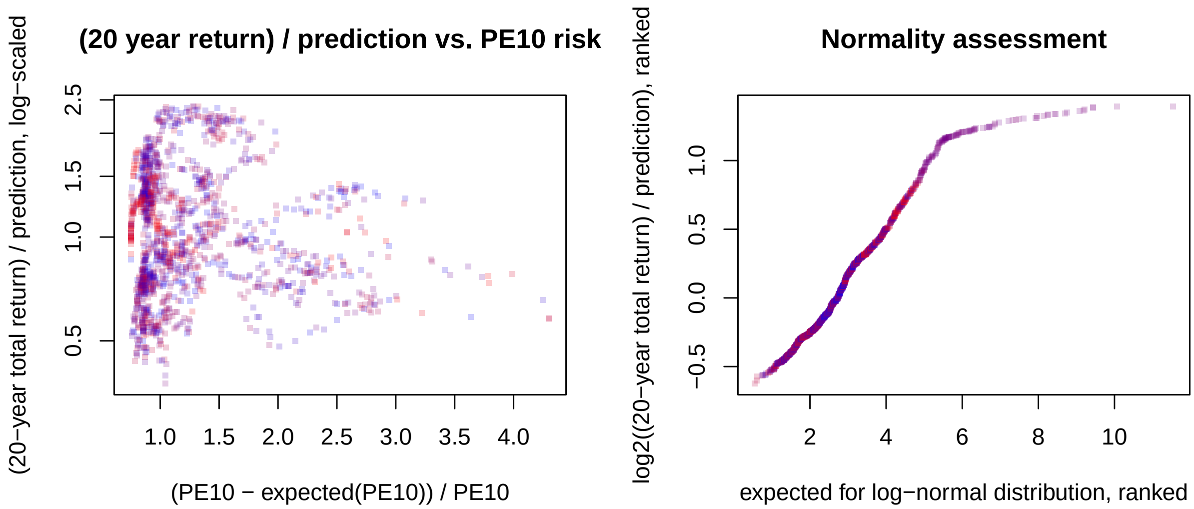
Hence:

$$\text{PE10-risk}_i \approx 1 - \frac{1.0}{\text{relative PE10}_i}$$

The next figure shows a relationship of percent 20-year total return to PE10-risk, with earlier points more red and later points more blue. The only difference between the plots is that the y-axis of the right-hand plot has been log-scaled. The prediction line is Tukey's robust line fitting, run for five iterations.



Dividing the 20-year returns by the fitted value gives residuals having little date-bias when risk is moderate-to-high (with the notable exception of most-elevated values, left panel of the figure below). However, ranking the log-transformed data shows pronounced non-normality for the highly ranked log-transformed risks (right panel).



Therefore, this risk-metric appears to have promise as a basis for an allocation strategy. Indeed, this is consistent with the argument that Shiller puts forth in *Irrational Exuberance*.

PE10-risk expressed in years

One may express PE10-risk in terms of how many years it takes to pay back the cost of buying stock when it is overpriced, relative to the alternative of leaving the money invested in bonds, as follows:

Define equity-to-bond risk premium as:

$$\text{risk-premium} = (\text{expected stock return}) - (\text{expected bond return}) = 6.44\% \text{ - } 2.02\% = 4.42\%/\text{year}$$

PE10-risk in years can then be expressed as:

$$\text{PE10-risk (years)} = \frac{\text{PE10-risk}}{\text{risk-premium}}$$

A risk-bounded buy-and-hold allocation strategy

This suggests that PE10-risk could be used to construct an allocation strategy (shown below as "Copeland's") that is basic buy-and-hold, trading only when risk-boundaries are reached:

Minimum equity allocation

$$\text{minimum allocation}_i = min\Big(1, max\big(0, K_{\text{low}} + (\text{expected relative PE10})_i - \text{PE10-risk}_i\big)\Big)$$
$$\approx min\big(1, max(0, K_{\text{low}} + 1.0 - \text{PE10-risk}_i)\big)$$

where i is the month and $K_{low} = 0.530154$.

Maximum equity allocation

The maximum equity allocation is inspired by Charles' approach (which is purely based on relative volatility; see below) because it dramatically reduces stock allocation once the relative PE10 increases past the 90th percentile (see the figure "percentile(relative PE10) vs. relative PE10" above.) However, it is computed relative to actual $\frac{E10}{P}$ and based on a maximum PE10-risk of 7.8 years (instead of Charles' 9.0 years).

$$\text{maximum allocation}_i = min\left(1, max\left(0, \frac{\frac{E10}{P} - 3.00\%}{4.03\% - 3.00\%}\right)\right)$$

Charles maximum equity allocation

The maximum equity allocation is taken from Charles' approach (which is purely based on relative volatility; see below) because it dramatically reduces stock allocation once the relative PE10 increases past the 90th percentile (see the figure "percentile(relative PE10) vs. relative PE10" above.)

$$\text{maximum allocation}_i = min\left(1, max\left(0, \frac{\text{9 years} - \text{PE10-risk(in years)}_i}{\text{9 years} - \text{3.3 years}}\right)\right)$$

Charles' minimum allocation

Charles' minimum allocation is analogously calculated:

$$\text{minimum allocation}_i = min\left(1, max\left(0, \frac{\text{6.9 years} - \text{PE10-risk(in years)}_i}{\text{6.9 years} - \text{0.81 years}}\right)\right)$$

Although Table A1 and Figure A1 in the Appendix use the exact formulas, the approximations seem quite sufficient.

Comparison of the historical performance of these allocation strategies

For comparison with Copeland's strategy:

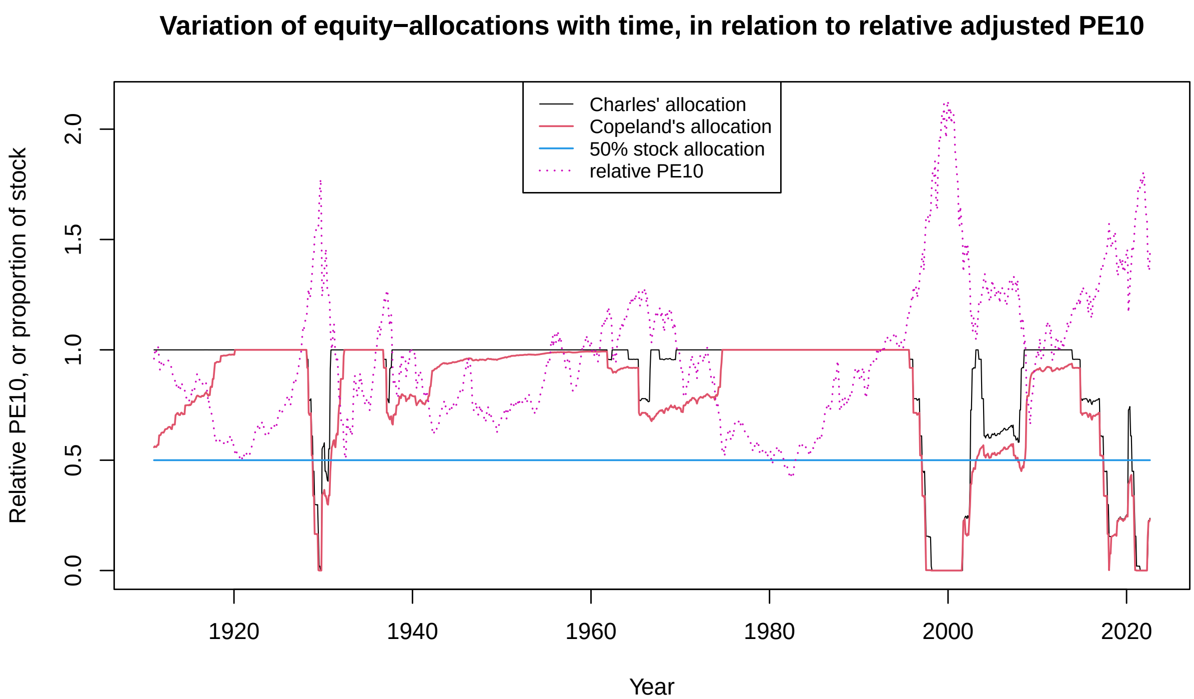
- Shown as "Charles"' is the strategy recevied from an individual inspired by Benjamin Graham and Warren Buffet, where his S&P500 boundaries have been converted to boundary adjusted-PE10. The minimum and maximum upper limits to the proportion of stock-allocation for the lower and upper boundaries are, respectively, 100.0% and 100.0%.
- Shown as "50% stock allocation" is a portfolio, rebalanced every month between 50% S&P 500 and 50% ten-year Treasury bonds.
- Shown as "100% stock allocation" is a portfolio including stocks (S&P 500) but not bonds.
- Shown as "0% stock allocation" is a portfolio including only ten-year Treasury bonds.

Of course, actual mutual funds incur expenses, so these results reflect a 0.07% annual expense ratio for bonds and a 0.04% annual expense ratio for stocks.

Figure – Variation of the investigated allocations with time

The next plot shows how the allocations investigated here vary with time; as a point of reference, also included in the plot is the "relative PE10", defined as follows:

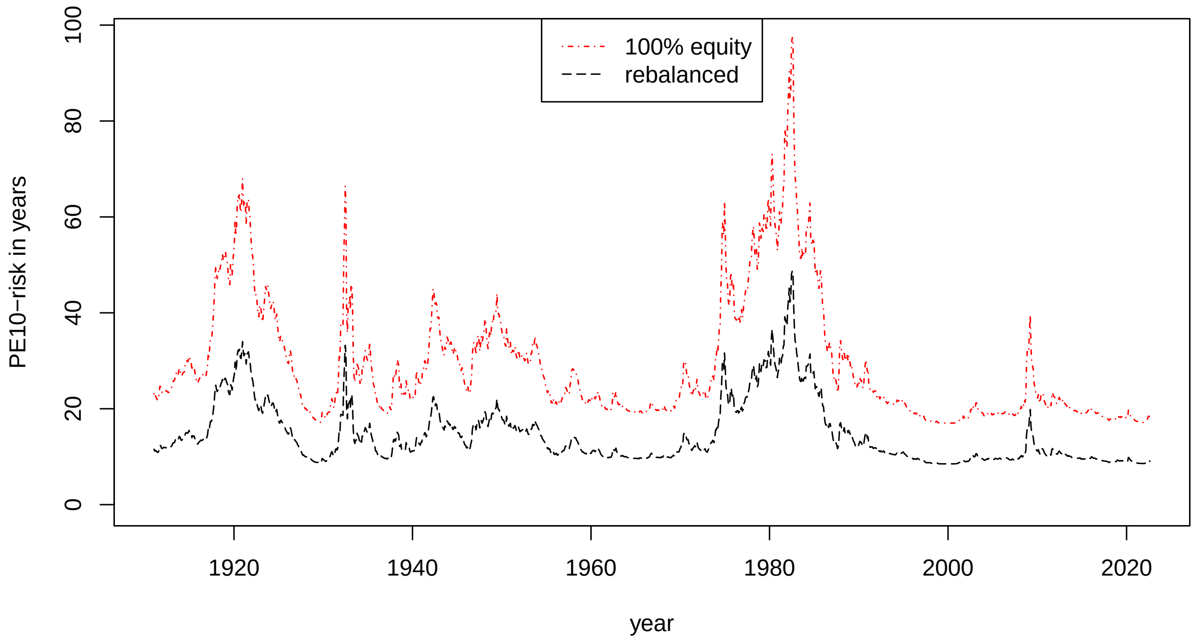
$$\text{relative PE10}_i = \frac{\text{adjusted-PE10}}{max(\text{PE10-trend}_i)}$$

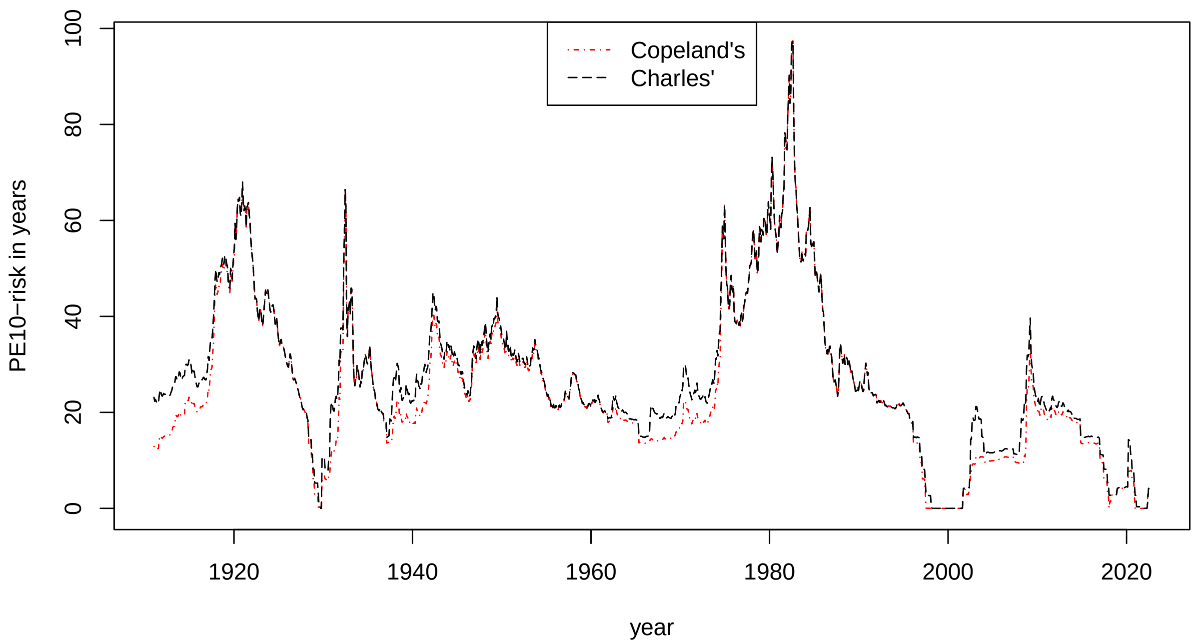


For each month i, and for each allocation strategy:

- The adjusted PE10 was used to locate the interval specifying the minimum and maximum allocation.
  - Notably, this is not how Charles' allocation is ordinarily performed; ordinarily, Charles performs an linear projection of the earnings expected divided by the current price to estimate the P/E. To a first approximation, this should also be smoothed to a value similar to the PE10, but it is not the same.
- If necessary, the percent equity was adjusted to bring the allocation within the allocation limits.
- Bond growth was computed as $\frac{\text{RealTotalBondReturn}_{1+i}}{\text{RealTotalBondReturn}_i}$ where $\text{RealTotalBondReturn}_i$ is defined in the Shiller spreadsheet as the total ten-year Treasury bond return as of month i in inflation adjusted dollars, assuming that all interest is re-invested.
- Stock growth was computed as $\frac{\text{TR-RealPrice}_{1+i}}{\text{TR-RealPrice}_i}$ where TR-RealPrice_i is defined in the Shiller spreadsheet as the total S&P 500 return as of month i in inflation adjusted dollars, assuming that all dividends and earnings are re-invested.
- The bond and stock holdings were adjusted according to their respective growths.
- The results for 0% and 100% stock portfolios were checked against those shown in the spreadsheet, as shown in Figure A5 in the Appendix.

Figure – PE10-risk in years for each strategy, versus the date.

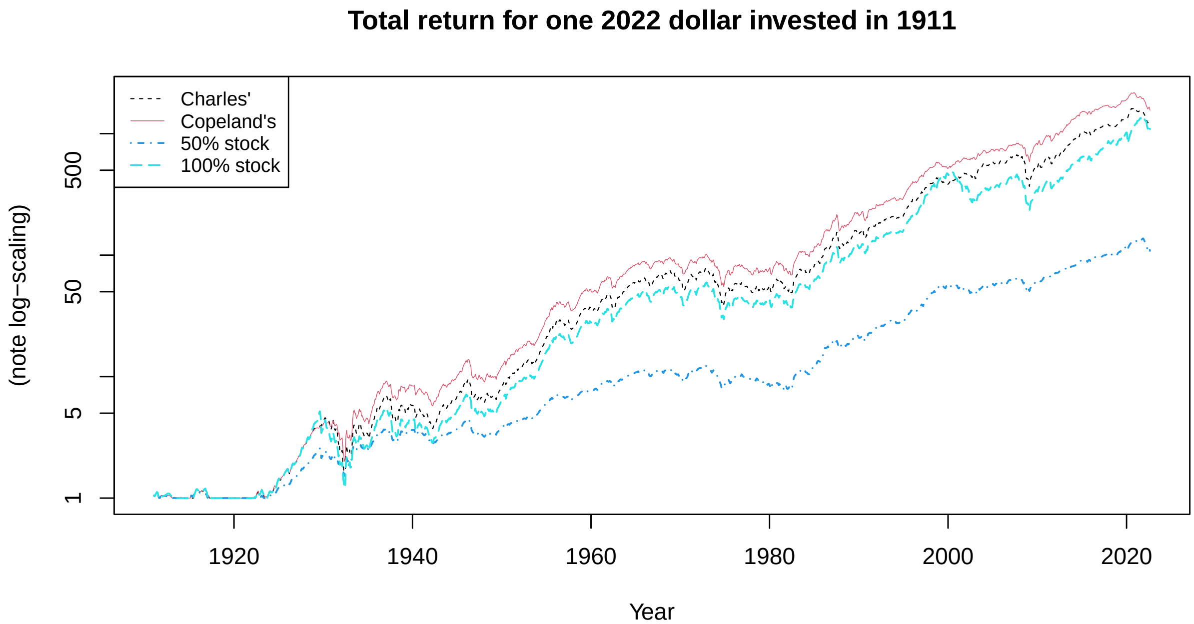




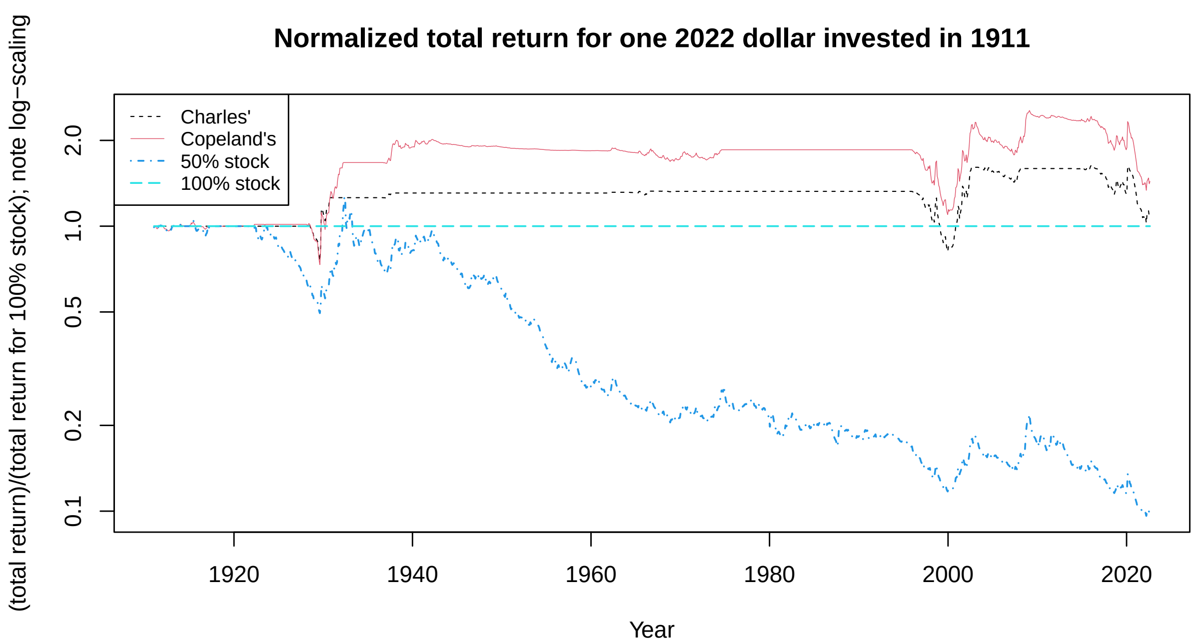
Total returns of the investigated allocation strategies

Figures – Total returns for each strategy.

The next few figures show that both Charles' and Copeland's allocation strategies greatly outperform the 100% equity allocation. Note that displacement along the Y-axis is proportional to log(total return) because total return grows geometrically.



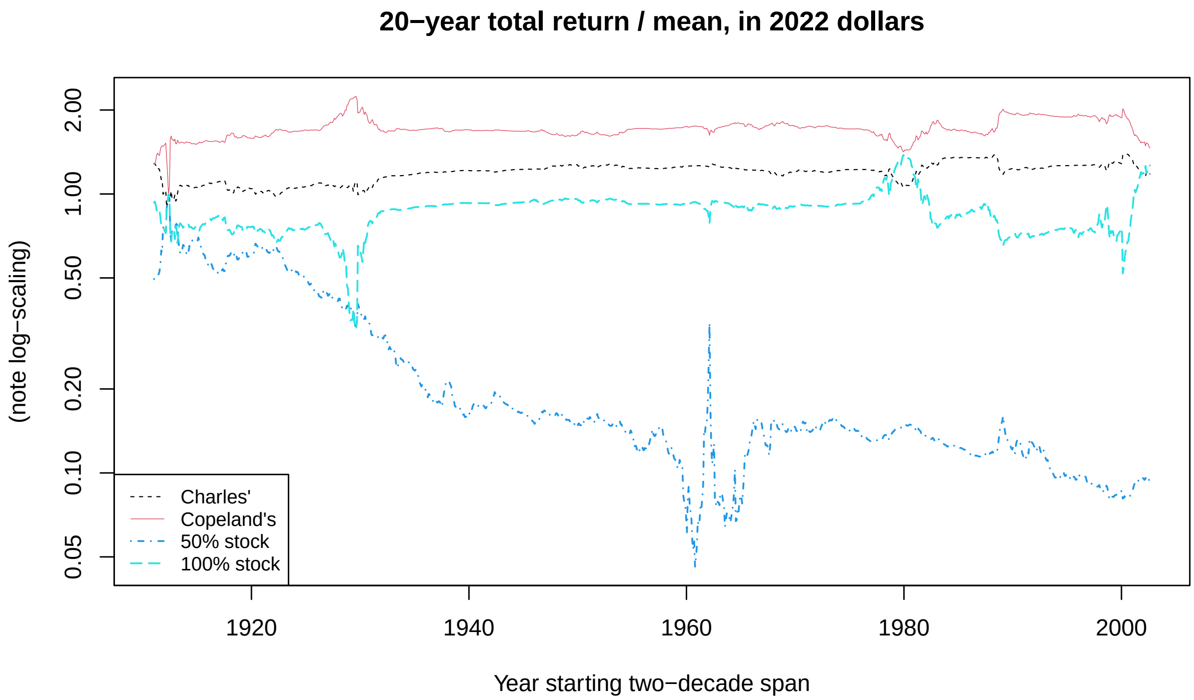
Normalizing by dividing by the return of the 100% stock portfolio can make it easier to see the relative effect of the several allocation strategies.



A 60%-balanced portfolio overall would have performed poorly because its equity-allocation is too low to keep pace with 100% stocks. Copeland's strategy would have favorably recovered from the dotcom bubble and would have been less sensitive to sharp declines in the price of stocks in 1929, 2008, 2020, and 2022.

Figures – Relative 20-year total returns for each strategy.

Total returns for 20-year periods starting in 1911 (relative to the mean for the four strategies) show that Charles' and Copeland's strategies would have consistently remained above the 50% stock allocation strategy; often, above the 100% stock allocation strategy.



Presented in the following table are total returns on $1.00 (2022 dollars) invested in 1911. To illustrate how much is gained by investing in stocks instead of bonds, a "risk premium" is computed and reported here. In colloquial terms, it shows how hard each equity dollar is working. The premium is computed as:

$$\text{strategy-to-bond risk premium} = \frac{\text{strategy percent return} - \text{bond percent return}}{\text{annual percent equity}} - 1$$

Table – Real returns since 1911 by allocation strategy

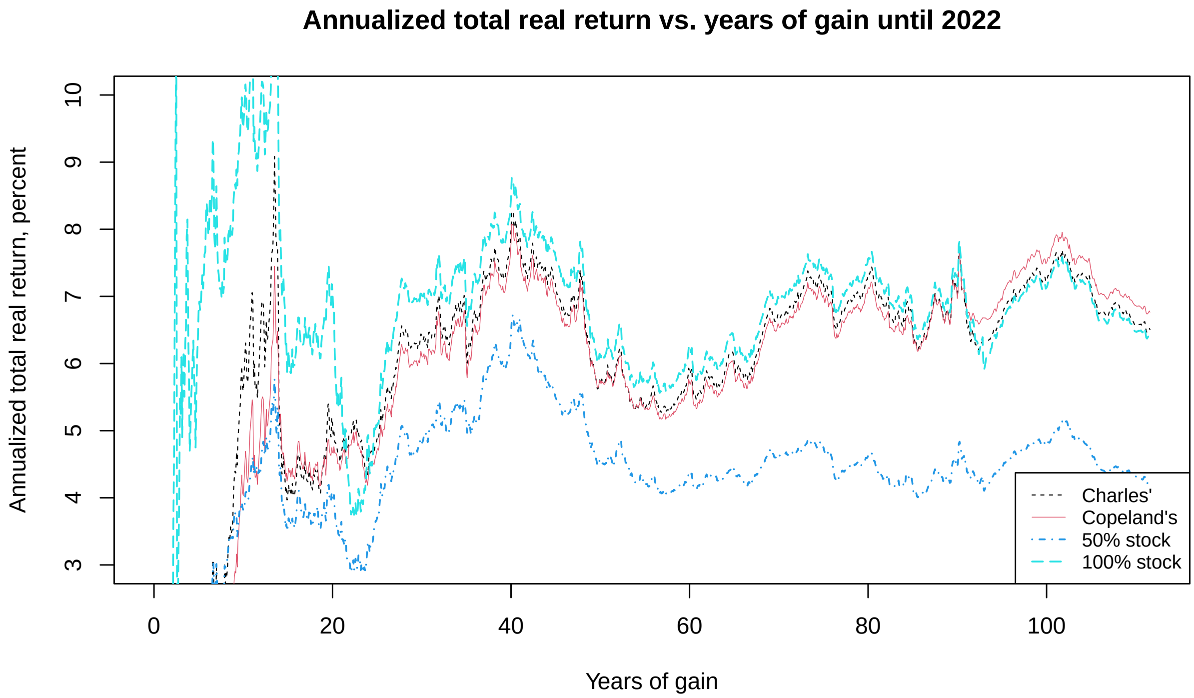
Allocation strategy	Total return	Annual growth	Mean % stock	Risk premium
Copeland's	$1562.33	6.81%	79.5%	563%
Charles'	$1200.12	6.56%	87.8%	472%
100% equity	$1088.06	6.47%	100.0%	393%
50% equity	$108.58	4.29%	50.0%	450%
0% equity	$5.49	1.54%	0.0%	

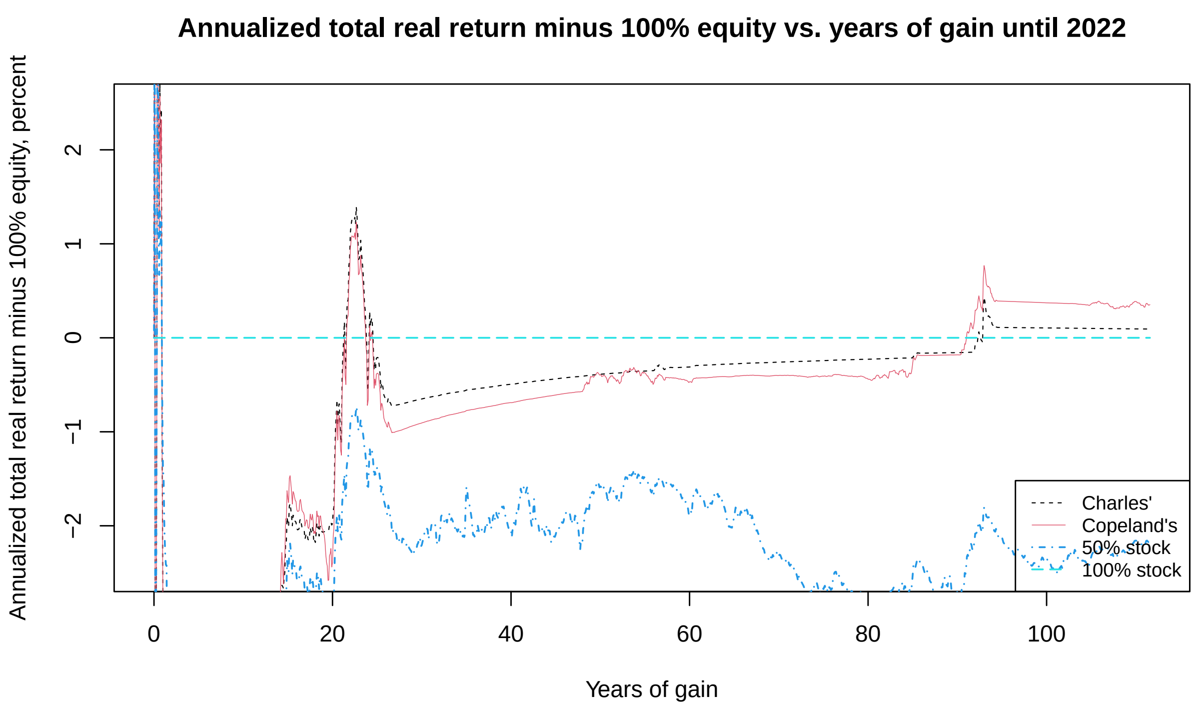
Note that increasing the 50%-stock portfolio to 85.8% would increase its growth rate to 5.82% per year at a risk premium of 429% (not demonstrated here), i.e., 91.5% of the 100%-stock portfolio, which is still more efficient than the latter but still produces less return than Copeland's or Charles' strategies.

Copeland's and Charles' strategies involve substantial allocation toward stocks except when PE10 is very high, and their gain would have been able able to exceed to the S&P 500. Copeland's strategy would have outperformed Charles' overall. Copeland's stock dollars would have been working the hardest (have the highest risk-premium); in the 100% equity allocation, stock dollars' showing would have been the weakest. By raising the upper boundary by a considerable amount (not demonstrated here), Charles' strategy could have delivered almost an additional 0.75% return per annum.

The previous table shows total real return from 1911 to 2022. The next figure shows the total real returns for periods ending in 2022. In the upper panel returns are shown directly; in the lower panel, after subtracting the return for the 100% equity strategy.

Figure – Annualized total real return vs. years of gain until 2022

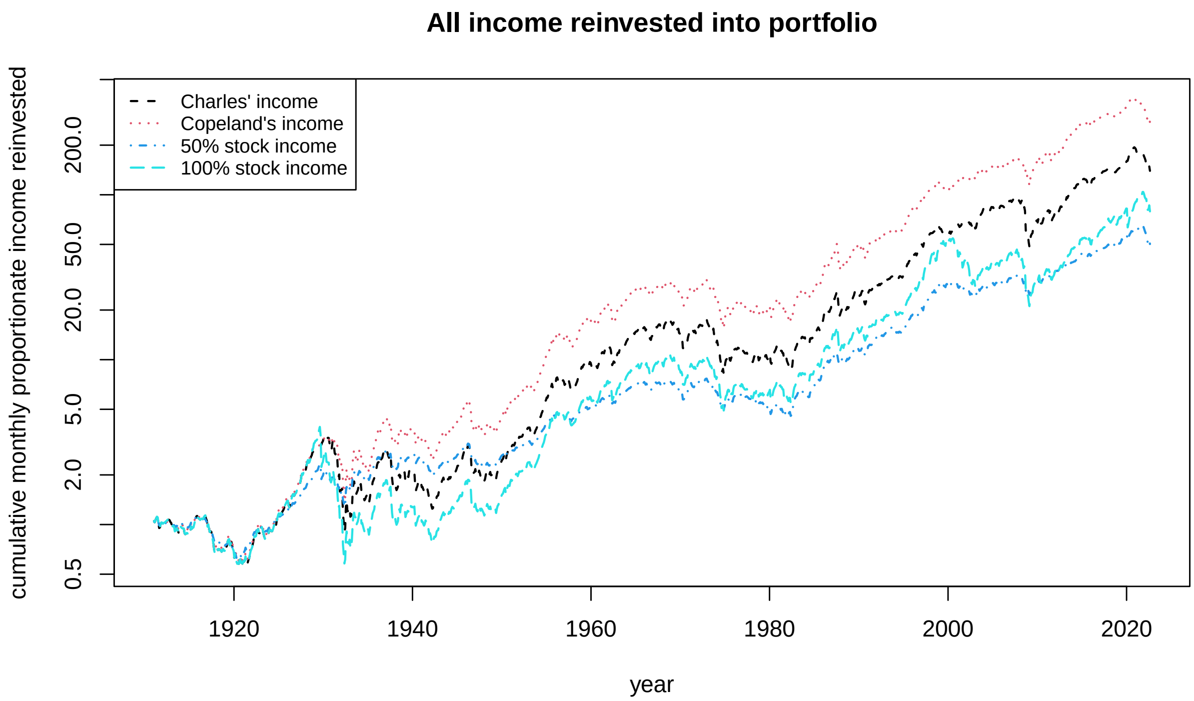


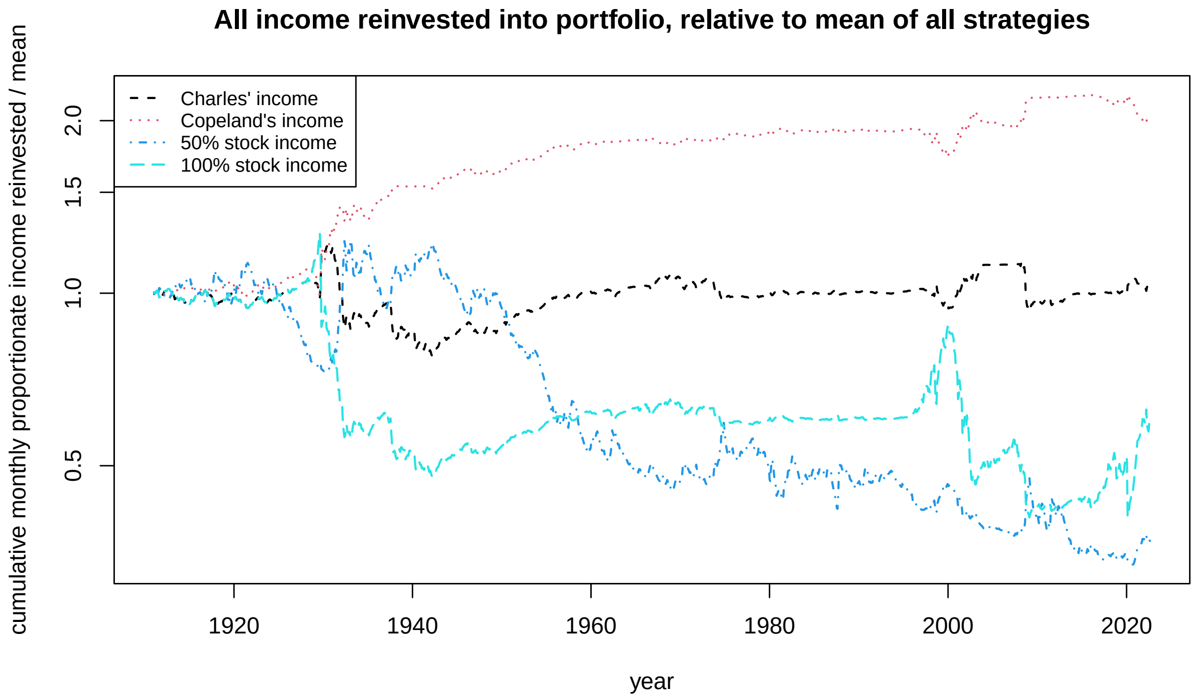


For periods greater than 20 years, Copeland's strategy usually returns more than the 100% equity strategy. I believe that the reason that Charles' strategy does not is because the upper boundary is set too conservatively.

Effect of allocation strategies on income

The preceding analyses have concerned what would have happened when all income was reinvested. However, in retirement, some (or, at times, most or all) income must be spent rather than invested (which is the point of investing, after all). The next plots show how these strategies would have performed regarding income generation. The first plot shows the performance of all four over time; the second, their performance relative to the mean of all four.

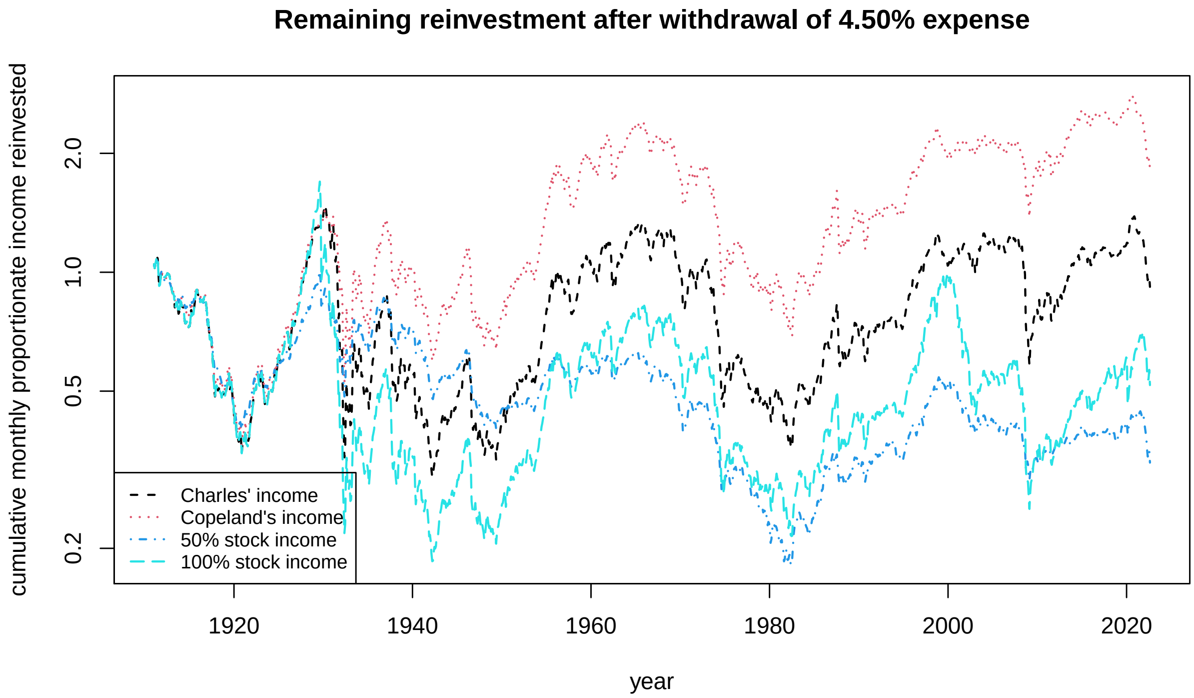


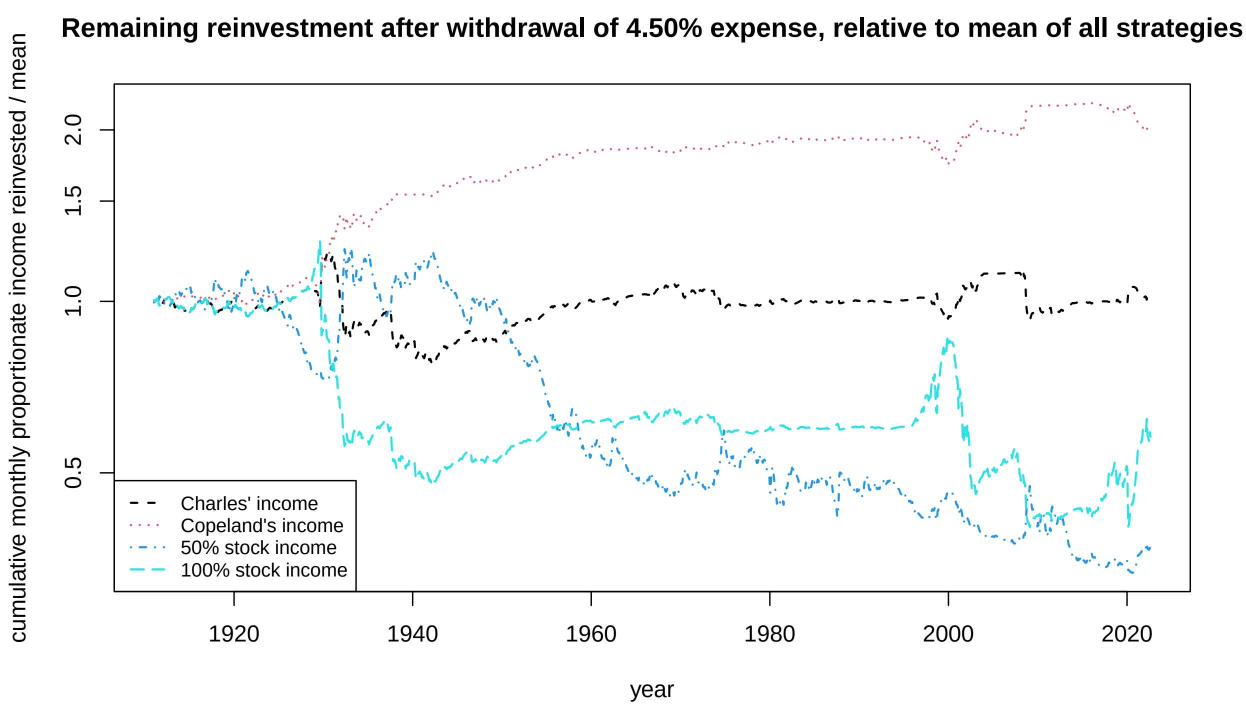


Copeland's strategy seems to produce the highest return, albeit slightly more than Charles'.

Effect of 4.50% annual expenses

The next plots show the effect an annual withdrawal rate of 4.50% from the portfolio.





At a 4.50% annual withdrawal rate, Charles' and Copeland's allocations would have shown little long-term decline in principal. By contrast, the other two (non-bond) allocations would have sufferred substantial declines in principal during the 1929, 2001, and 2008 financial criss.

Not shown here: The 100% stock and 50% stock strategies would have needed a 3.75% annual withdrawal rate to reduce long-term decline.

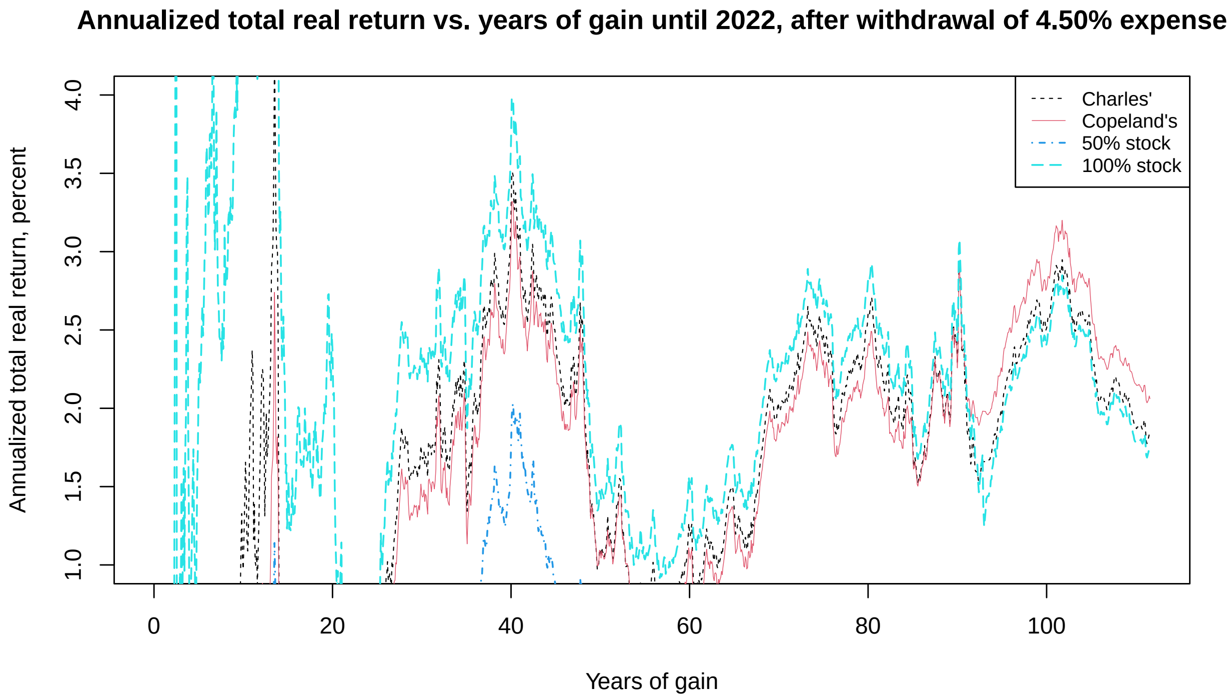
Table – Real returns since 1911 by allocation strategy, with 4.50% expenses

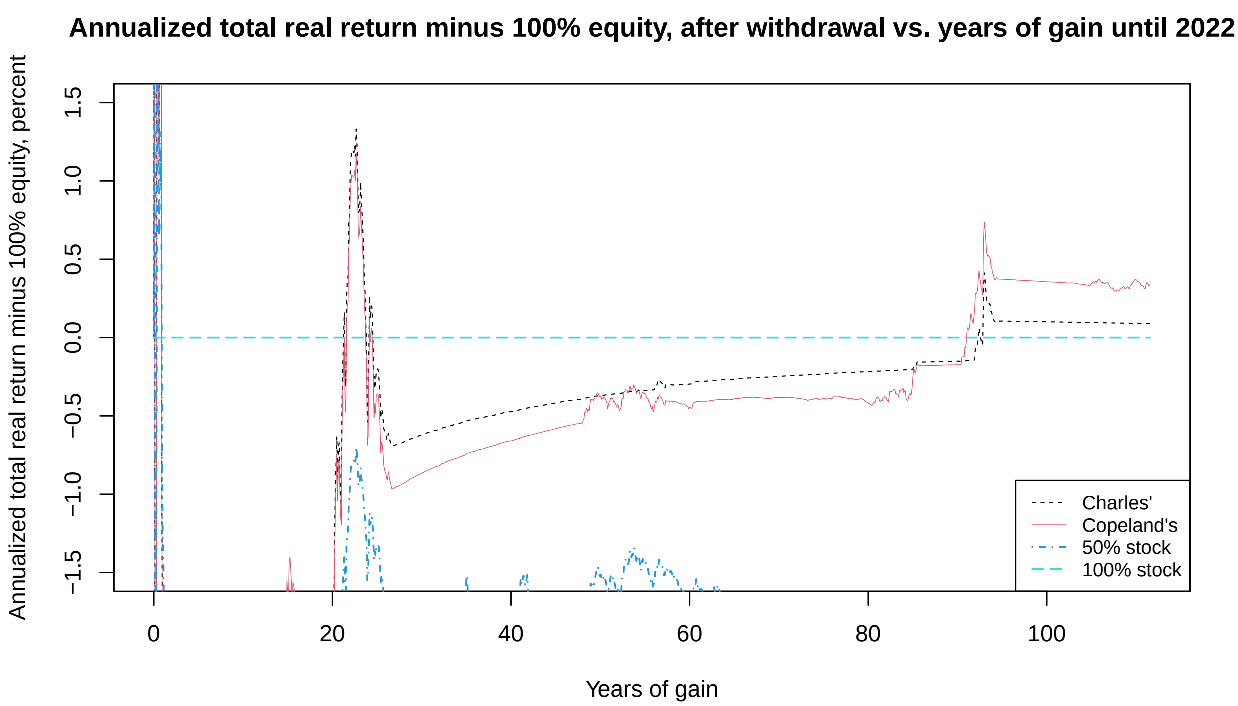
Allocation strategy	Total return	Annual growth	Mean % stock	Risk premium
Copeland's	$10.17	2.10%	79.5%	534%
Charles'	$7.81	1.86%	87.8%	447%
100% equity	$7.08	1.77%	100.0%	371%
50% equity	$0.71	-0.31%	50.0%	426%
0% equity	$0.04	-2.94%	0.0%	

Again, Charles' strategy narrowly *but meaningfully* would have outperformed 100% stock; Copeland's would have achieved the greatest risk-premium when expenses are taken into account.

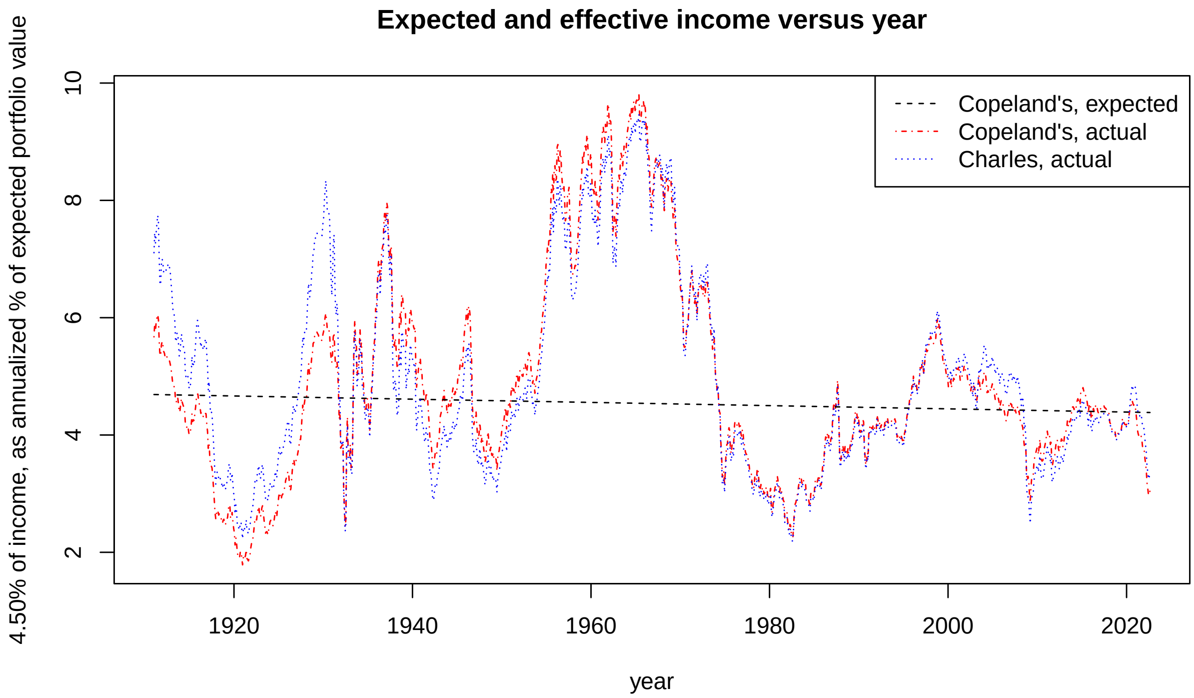
The next figures show that Copeland's strategy generally would have kept the income withdrawn the highest among the strategies.

Figure – Annualized total real return vs. years of gain until 2022, with 4.50% annual expenses





The next figure shows that Charles' strategy would have required a slightly smaller cash reserve than Copeland's to buffer against transiently diminished income.



Concluding thoughts

Given the premises of this investigation, results for Copeland's and Charles' allocation strategies show that the 50% equity allocation is far too conservative. Although these results demonstrate the effect of allocation on the sum of the total returns for both S&P 500 and 10-year Treasury bonds, the yield of the bonds was not part of the risk estimation.

Copeland's estimated minimum and maximum PE10-risks were determined empirically, and risk estimation considered mostly the risk of the PE10 reverting to the trend rather than volatility (although the latter was indeed in effect when testing th strategies). The values used in this investigation for these boundaries were chosen as "local maxima" (alternately holding one constant and adjusting the others for maximum total return for 1911-2022). Two cautionary notes are in order:

- Only minimal attempts were made to find some other combination that might outperform them.
- They are completely dependent on the historical data and unproven for the future.

Responding to measurable risk and opportunity

Do Copeland's and Charles' approaches constitute market timing? I believe that the answer to this question is simply "no", but everyone denies that they are market timing; you must be the judge whether responding to measurable risk is "market timing". The graph of the proportion of the portfolio allocated to equities shows that 87.5% of the time Copeland's allocation is above 50% equity; 59.5% of the time, above 86% equity. I maintain that this is effectively a risk-moderated buy-and-hold strategy.

The risk boundaries limit opportunity lost to under-allocation and value lost to *extreme* over-allocation. *Trading is required only when these boundaries are crossed;* because they are widely spaced, trading should be relatively rare except during times of considerable movement when stocks are considerably overvalued. *Only* *when prices reverse their trend at extreme rates* is it necessary to switch between increasing and decreasing allocation. Trades cost attention, time, and sometimes money; having fewer trades to make reduces these burdens.

Why does the 60% stock portfolio perform relatively poorly?

Commonly available (to me) investment advice has been to “rebalance the portfolio” when stock prices undergo considerable change. Why would this be the recommendation? Why would our fund managers have told us, ‘The research that has been done has shown that safe withdrawal rate of 5% can be sustained by a portfolio that is 60% bonds and 40% stocks.’“? Investment products such as the Vanguard Balanced Index Index Fund exist to provide zero-effort maintenance of a 60% stock / 40% bond fund; why would this be so?

I believe that this advice targets the very conservative investor who nervously eyes their portfolio as stock prices go down (“Should I trade my stocks for bonds before stocks go lower?”) or go up (“Should I trade my bonds for stocks now before the opportunity eludes me?”). It gives a very simple discipline to follow without the anxiety of stock prices falling substantially soon before a big non-investment expense is incurred by the investor. It is supposed to buy peace-of-mind.

However, the 60% portfolio only has 60% of the capacity to keep pace with returns from the stock market, making the 60% limit difficult to justify when stocks are not substantially overvalued. This conclusion is supported by the demonstration reported here.

What is the effect of rebalancing on returns? French (2021) finds that it diminishes returns.

What about Shiller/Bunn “Total Return CAPE”?

Bunn and Shiller (2014) introduced a refinement of the PE10 that is termed the “Total Return CAPE” which treats dividend pay-outs and stock buy-backs uniformly. The analysis presented here does not differ substantially when the TR CAPE is substituted for the PE10. Because computation of the TR CAPE is somewhat more complex, and because the PE10 is commonly used and relatively easily computed, the results with the latter are presented here.

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Appendix

Allocation profiles for 2023

Here are the allocation profiles presented in graphical and tabular forms. S&P 500 prices, P/E, and adjusted-PE10 are for 2023. Note that

$$percentile(\text{relative PE10})/100 = \frac{rank(\text{relative PE10})}{count(\text{relative PE10})}$$

represents the historical probability that the relative PE10 would have been less than the value shown on the X axis. (Statisticians will recognize this a the cumulative distribution for relative PE10).

The next plot:

- presents the allocation minima and maxima that are presented in Table A1 below,
- also adds the $percentile(\text{relative PE10})/100$ defined immediately above.

Figure A1 - Allocation profile graph for 2023

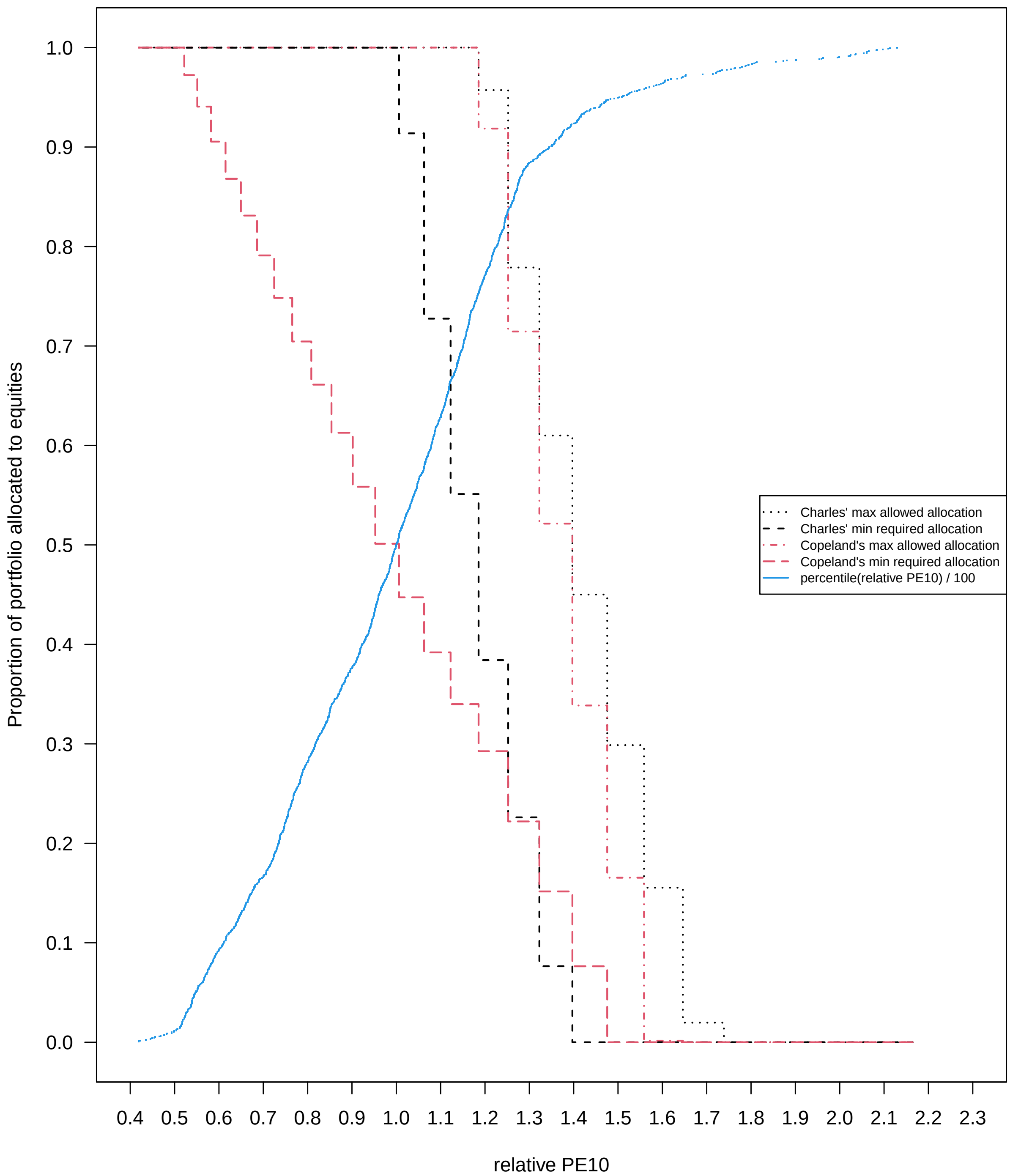


Table A1 – Allocation profiles for 2023

S&P500 prices in the following table are for 2023.

PE10 in the following table is the adjusted-PE10, i.e., the PE10 after adjustment for the trend through 2022, as described above, i.e.:

$$\text{adjusted-PE10}_i = \text{PE10}_i + \text{expected adjusted-PE10} - \text{PE10-trend}_i$$

$$\text{expected adjusted-PE10} = max(\text{PE10-trend}) = 21.3748745 \text{ for September 2022.}$$

$$\text{PE10-trend}_i = (0.0578777)(\text{DateFraction}) \text{ - } 95.694931$$

$$\text{DateFraction} = \text{year} + \frac{i-1}{12}$$

i is month number.

SP500	PE	PE10	risk	years	PE10 percentile	relative PE10	Co min	Co max	Co max risk	Ch min	Ch max	Ch max risk
6371	27.34	46.26	0.54	12.2	100.0	2.16	0.00	0.00	0.00	0.00	0.00	0.00
6031	25.88	43.80	0.51	11.5	99.4	2.05	0.00	0.00	0.00	0.00	0.00	0.00
5710	24.51	41.47	0.48	10.9	98.8	1.94	0.00	0.00	0.00	0.00	0.00	0.00
5406	23.20	39.26	0.45	10.2	98.5	1.84	0.00	0.00	0.00	0.00	0.00	0.00
5118	21.97	37.17	0.42	9.4	97.7	1.74	0.00	0.00	0.00	0.00	0.00	0.00
4846	20.80	35.19	0.38	8.7	97.1	1.65	0.00	0.00	0.00	0.00	0.02	0.01
4588	19.69	33.32	0.35	7.8	95.8	1.56	0.00	0.00	0.00	0.00	0.16	0.05
4343	18.64	31.54	0.31	7.0	94.6	1.48	0.08	0.17	0.05	0.00	0.30	0.09
4112	17.65	29.86	0.27	6.1	92.2	1.40	0.15	0.34	0.09	0.08	0.45	0.12
3893	16.71	28.27	0.23	5.3	89.2	1.32	0.22	0.52	0.12	0.23	0.61	0.14
3686	15.82	26.77	0.19	4.3	83.6	1.25	0.29	0.71	0.14	0.38	0.78	0.15
3490	14.98	25.34	0.16	3.6	75.3	1.19	0.34	0.92	0.15	0.55	0.96	0.15
3304	14.18	23.99	0.12	2.8	66.5	1.12	0.39	1.00	0.12	0.73	1.00	0.12
3128	13.42	22.71	0.08	1.8	57.8	1.06	0.45	1.00	0.08	0.91	1.00	0.08
2961	12.71	21.51	0.03	0.7	50.9	1.01	0.50	1.00	0.03	1.00	1.00	0.03
2804	12.03	20.36	0.00	0.0	43.6	0.95	0.56	1.00	0.00	1.00	1.00	0.00
2654	11.39	19.28	0.00	0.0	37.8	0.90	0.61	1.00	0.00	1.00	1.00	0.00
2513	10.79	18.25	0.00	0.0	33.8	0.85	0.66	1.00	0.00	1.00	1.00	0.00
2379	10.21	17.28	0.00	0.0	29.0	0.81	0.70	1.00	0.00	1.00	1.00	0.00
2253	9.67	16.36	0.00	0.0	24.2	0.77	0.75	1.00	0.00	1.00	1.00	0.00
2133	9.15	15.49	0.00	0.0	18.8	0.72	0.79	1.00	0.00	1.00	1.00	0.00
2019	8.67	14.66	0.00	0.0	16.0	0.69	0.83	1.00	0.00	1.00	1.00	0.00
1912	8.20	13.88	0.00	0.0	12.9	0.65	0.87	1.00	0.00	1.00	1.00	0.00
1810	7.77	13.14	0.00	0.0	10.4	0.61	0.91	1.00	0.00	1.00	1.00	0.00
1713	7.35	12.44	0.00	0.0	7.8	0.58	0.94	1.00	0.00	1.00	1.00	0.00
1622	6.96	11.78	0.00	0.0	5.3	0.55	0.97	1.00	0.00	1.00	1.00	0.00
1536	6.59	11.15	0.00	0.0	2.4	0.52	1.00	1.00	0.00	1.00	1.00	0.00
1454	6.24	10.56	0.00	0.0	1.0	0.49	1.00	1.00	0.00	1.00	1.00	0.00
1377	5.91	10.00	0.00	0.0	0.7	0.47	1.00	1.00	0.00	1.00	1.00	0.00
1303	5.59	9.46	0.00	0.0	0.3	0.44	1.00	1.00	0.00	1.00	1.00	0.00
1234	5.30	8.96	0.00	0.0	0.1	0.42	1.00	1.00	0.00	1.00	1.00	0.00

Figure A2 - PE10-risk (in years) graph for allocation strategies

This figure shows the expected number of years that it would take for a given allocation to make up the difference after buying US large-cap stock when it is too expensive.

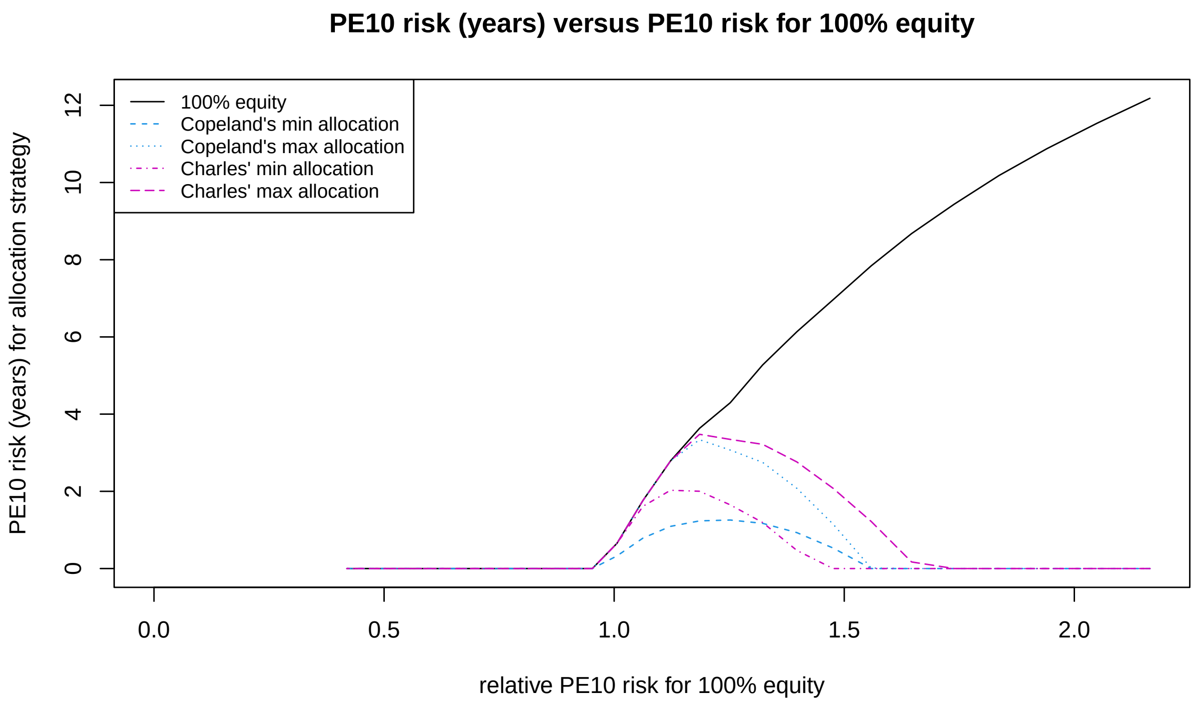


Figure A3 - Expected relative-PE10 vs. relative-PE10

Given a particular PE10 observed over the period 1881–2022, what is the statistically expected loss of its value, expressed as a proportion, over this entire period? This may be computed through a sequence of fairly straightforward steps:

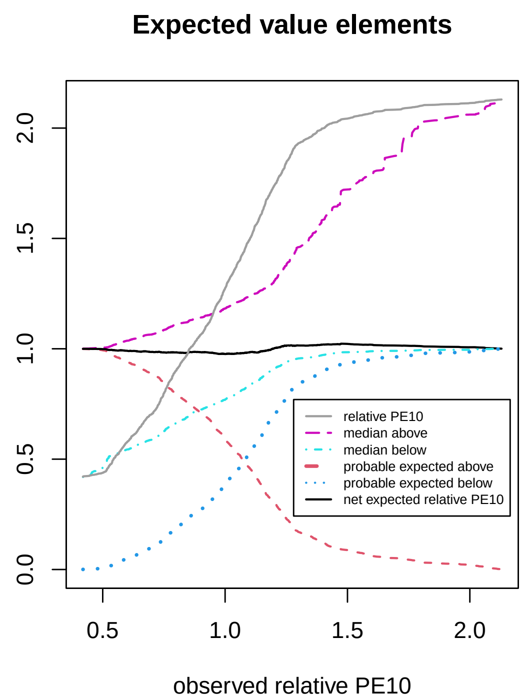
In the left panel of the next figure, the grey curve labeled “relative rank(PE10)” shows:

$$\text{relative rank(PE10)} = \frac{rank(\text{PE10})}{count(\text{PE10})}$$

relative rank(PE10) does not have the gently curving “S” shape expected for a normal distribution. Therefore, using the the median to estimate the expected value is a more suitably robust alternative to using the mean.

- The median of relative PE10 values whose rank is below the rank of “relative rank(PE10)” is shown in magenta with the long-dashed curve, labeled “median below”.
- The “relative rank(PE10)” is the probability that the relative PE10 will be below the observed relative PE10.
- Thus, the “relative rank(PE10)” times the “median below” is the “weighted” contribution to the “net expected PE10” of low values, the short-dashed red curve in the figure, labeled “probable expected below”.
- The median of relative PE10 values whose rank is above the rank of “relative rank(PE10)” is shown in cyan with the dot-dash curve, labeled “median above”.
- One minus the “relative rank(PE10)” is the probability that the relative PE10 will be above the observed relative PE10.
- The “probable expected above”, that is the product of the “median above” and the previous two values, shown as the dark blue dotted curve, labeled “probable expected above”.
- The “net expected relative PE10”, shown as a solid black curve, is the sum of the “probable expected below” and the “probable expected above”, i.e., the dotted dark blue curve and the dashed red curve.

The actual expected relative PE10 is within ±2.5% of the overall median expected value (right panel).


Expected value elements
relative PE10
median above
median below
probable expected above
probable expected below
net expected relative PE10
observed relative PE10
0.0
0.5
1.0
1.5
2.0

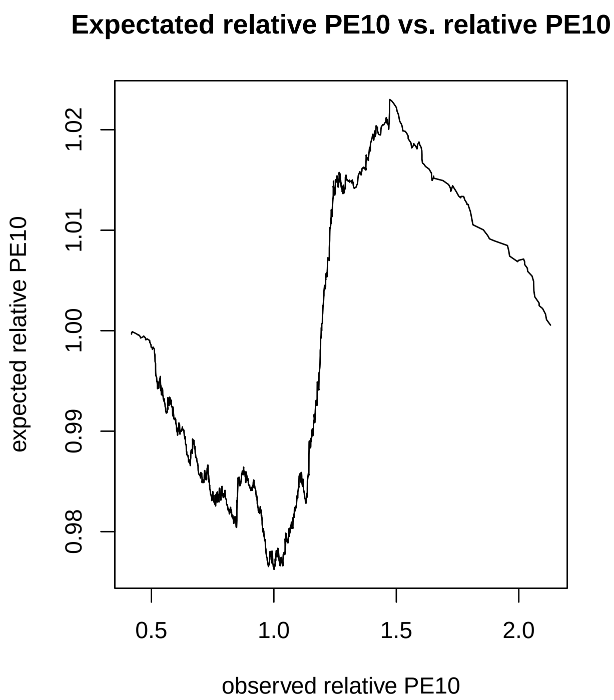

Expectated relative PE10 vs. relative PE10
expected relative PE10
observed relative PE10
0.98
0.99
1.00
1.01
1.02
0.5
1.0
1.5
2.0

Figure A4 - Expected proportionate change vs. relative-PE10

The expected proportionate change for an observed PE10 is given by

$$\text{expected proportionate change} = 1 - \frac{\text{expected relative PE10}}{\text{observed relative PE10}}$$

This is plotted vs. the relative PE10 in the next figure. The asymmetry of this curve is due to the asymmetry of the "relative rank(PE10)" curve; the scatter in the left tail, due to variability in the left tail of the "relative rank(PE10)" curve.

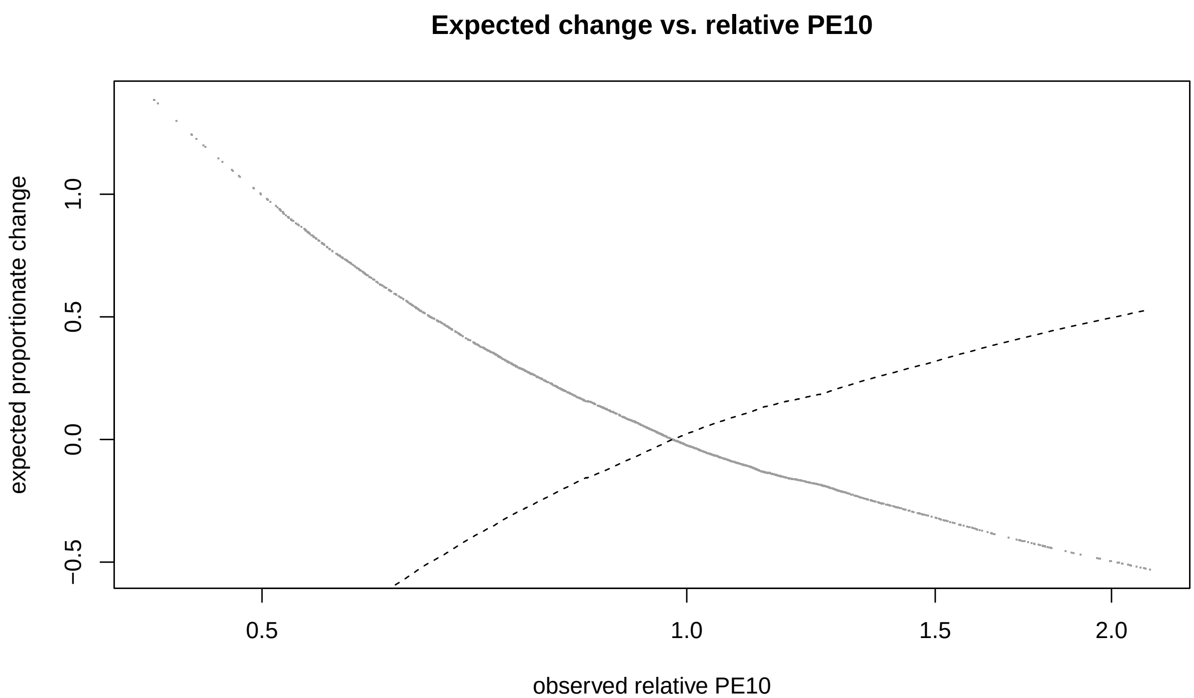
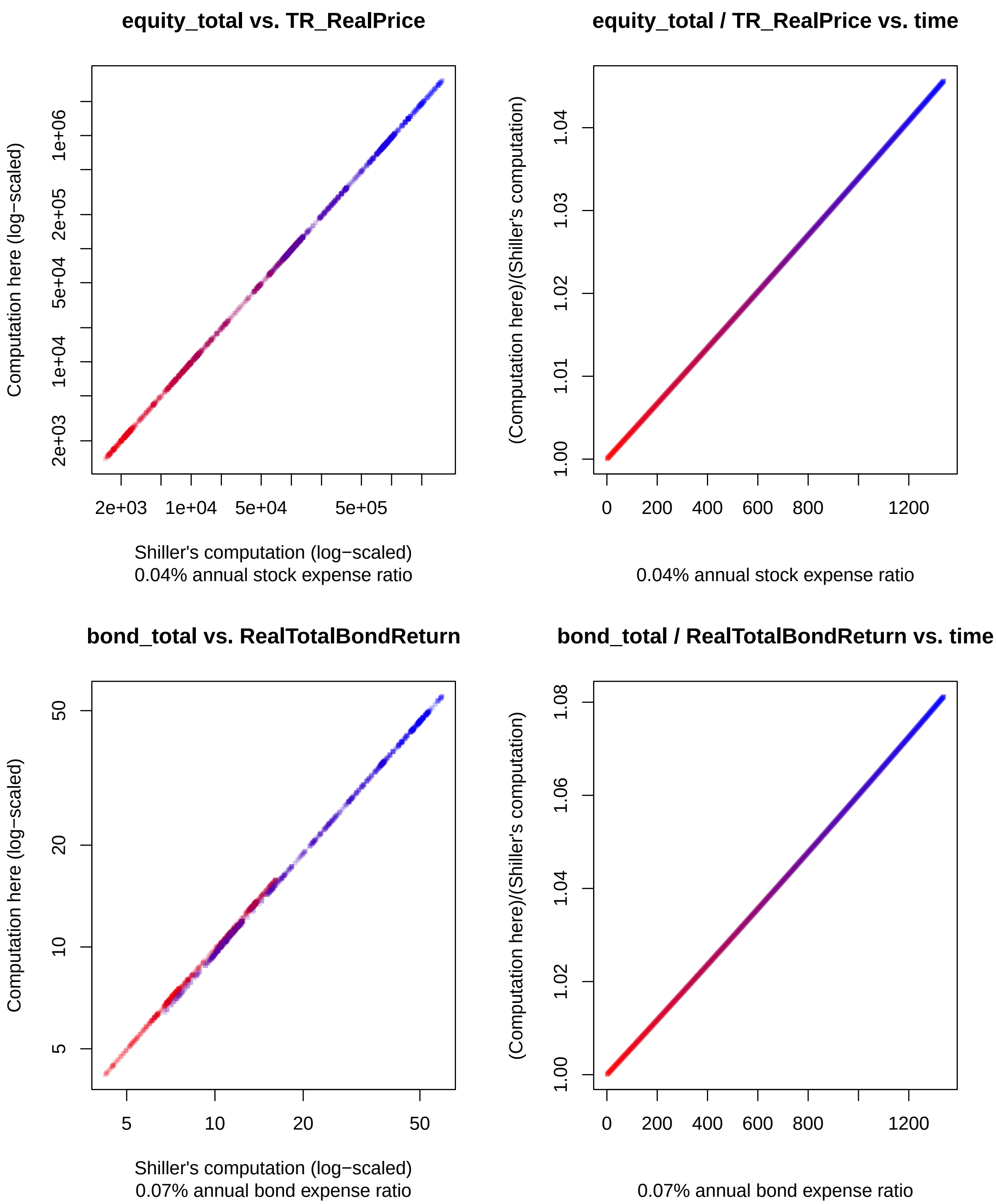


Figure A5 - quality checks for `trade_and_grow`



Peculiar relationship between PE10 and stock total returns to date

Although I have no explanation, there is a great degree of correlation ($r^2 = 0.9955$) between to total return from stocks and a peculiar combination of the trend in the PE10 and the trend in the total return (Shiller's "Total Return Stock Price") at time i:

$$(5 \times 10^{-7})(\text{trend-adjusted PE10}_i)(\text{stock price trend}_i)^{3.5} \approx \text{stock price}_i$$

Figure A6 - Observed relationship of munged PE10 and stock price